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An Application Of Rough Topology In Foot And Mouth Disease Of The Animal

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ABSTRACT: Thispaper presents the study of new structure in rough topology. The aim of this studyare to examine the rule of potential risk factors for some disease. Using the concept of rough topology, we find the deciding factors for the most common Foot and Mouth disease in Animal.

Keywords:Rough sets, Rough topology, Upper Approximation, Lower Approximation, Foot and Mouth disease.

1. INTRODUCTION

The concept of Rough Sets (RS) was proposed by Zdzisław Pawlak in 1982 [6], but their origins are in his previous work from 1981, when he introduced the rough relations, the classification of objects by attributes and information systems.The main idea behind RS is that an amount of information is associated to each object from the universe of discourse.The theory is based on the concept of information system, which is a tabularized data set. The columns are labeled as "attributes", while rows are labeled as "objects" or "events".This theory deals with the approximation of sets or concepts by means of equivalence relations and is considered as one of the first non-statistical approaches in data analysis. Several interesting applications of the theory have come up, in particular, in Artificial Intelligence and Cognitive Sciences. The main difference between rough sets and fuzzy sets is that the rough sets have precise boundaries whereas fuzzy set theory is generally based on ill-defined sets of data, where the bounds are not precise and hence fuzzy predictions tend to deviate from exact values. The lower and upper approximations of a set are analogous to the interior and closure operations in a topology called rough topology in terms of lower and upper approximations of a rough set and we have applied the concept of topological basis to find the deciding factor for Foot and Mouth disease of the animal.

2. PRELIMINARIES

Definition 2.1[6]:Let U be a non-empty finite set of objects called the **universe** and R be an equivalence relation on U named as indiscernibility relation. The pair (U, R) is called the **approximation space**. Let X be a subset of U.

i) The **lower approximation** of X with respect to R is the set of all objects, which can be certain classified as X with respect to R and is denoted by $R_*(X)$.That is, $R_*(X) =$

$\bigcup_{x \in U} \{ R(X) : R(X) \subseteq X\}$ where $R(X)$ denotes the equivalence class determined by x.

ii) The **upper approximation** of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is denoted by $R^*(X)$.That is, $R^*(X) = \bigcup_{x \in U} \{ R(X) : R(X) \cap X \neq \emptyset \}$.

iii) The **boundary region** of $X$with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and is denoted by $B_R X$. That is, $B_R(X) = R^*(X) - R_*(X)$

The set X is said to be **rough** with respect to R if $R^*(X) \neq R_*(X)$.That is, if $B_R(X) \neq \emptyset$.

Preposition 2.2[6]:If (U, R) is an approximation space and X and Y are subsets of U,then

i) $R_*(X) \subseteq X \subseteq R^*(X)$
ii) $R_*(\emptyset) = R^*(\emptyset) = \emptyset$ and $R_*(U) = R^*(U) = U$
iii) $R^*(X \cup Y) = R^*(X) \cup R^*(Y)$
iv) $R_*(X \cup Y) \supseteq R_*(X) \cup R_*(Y)$
v) $R_*(X \cap Y) = R_*(X) \cap R_*(Y)$
vi) $R^*(X \cap Y) \subseteq R^*(X) \cap R^*(Y)$
vii) $R_*(X) \subseteq R_*(Y)$ and $R^*(X) \subseteq R^*(Y)$ whenever $X \subseteq Y$
viii) $R_*(X^C) = [R^*(X)]^C$ and $R^*(X^C) = [R_*(X)]^C$
ix) $R_*R_*(X) = R^*R_*(X) = R_*(X)$
x) $R^*R^*(X) = R_*R^*(X) = R^*(X)$

Remark 2.3:$R^*: P(U) \to P(U)$ satisfies the Kuratowski closure axioms that

i) $R^*(\emptyset) = \emptyset$
ii) $X \subseteq R^*(X)$
iii) $R^*(X \cup Y) = R^*(X) \cup R^*(Y)$
iv) $R^*R^*(X) = R^*(X)$ for all subsets X and Y of U

If $F = \{X \subseteq U / R^*(X) = X\}$, using conditions $(i)$and (iv), we see that $\emptyset$ and U are in F; $X \cup Y \in F$ whenever $X and\ Y$ are in $F\ and \bigcap X_\alpha \in F$ for all X_α in F. Therefore, the family T, of complements of members of F is a topology on U. Thus , F is the family of T-closed sets. Also, $Cl(X) = R^*(X)$.Therefore,R^* is the Kuratowski closure operator.

Remark 2.4:Since $R^*: P(U) \to P(U)$ satisfies the following properties that

i) $R^*(U) = U$
ii) $R_*(X) = X$
iii) $R_*(X \cap Y) = R_*(X) \cap R_*(Y)$
iv) $R_*R_*(X) = R_*(X)$ for all subsets X and Y of U,the operator R_* is the interior operator.

3.ROUGH TOPOLOGY

In this section we introduce a new topology called rough topology in terms of lower and rough approximations.

Remark 3.1:Let U be the universe of objects and R be an equivalence relation on U.
For $X \subseteq U$,we define $\tau_R = \{ U , \emptyset , R_*(X) , R^*(X) , B_R(X)\}$, where $R^*(X)$, $R_*(X)$ and $B_R(X)$ are respectively the upper approximation, lower approximation and the boundary region of X with respect to R.We note that U and $\emptyset \in \tau_R$.Since, $R_*(X) \subseteq R^*(X)$, $R_*(X) \cup R^*(X) = R^*(X) \in \tau_R$. Also, $R^*(X) \cup B_R(X) = R^*(X) \in \tau_R$ and $R_*(X) \cup B_R(X) = R^*(X) \in$

$\tau_R(X)$. Also, $R_*(X) \cap R^*(X) = R_*(X) \in \tau_R$; $R^*(X) \cap B_R(X) = B_R(X) \in \tau_R$ and $R_*(X) \cap B_R(X) = \emptyset \in \tau_R$.

Definition 3.2: Let U be the universe, R be an equivalence relation on U and $\tau_R = \{U, \emptyset, R_*(X), R^*(X), B_R(X)\}$ where $X \subseteq U$. τ_R satisfies the following axioms:

i) U and $\phi \in \tau_R$.
ii) The union of the elements of any subcollection of τ_R is in τ_R.
iii) The intersection of the elements of any finite subcollection of τ_R is in τ_R.

τ_R forms a topology on U called as the rough topology on U with respect to X. We call (U, τ_R, X) as the **rough topological space**.

Example 3.3: Let $U = \{a, b, c, d, e\}$, $U / R = \{\{a, b\}, \{c, d\}\{e\}\}$, the family of equivalence class of U by the equivalence relation R and $X = \{a, c, d\}$. Then, $R^*(X) = \{a, b, c, d\}$, $R_*(X) = \{c, d\}$ and $B_R(X) = \{a, b\}$. Therefore, the rough topology $\tau_R = \{U, \emptyset, \{a, b, c, d\}, \{c, d\}, \{a, b\}\}$.

Preposition 3.4: If τ_R is the rough topology on U with respect to X, then the set $B = \{U, R_*(X), B_R(X)\}$ is the basis for τ_R.

Proof:

i) $\bigcup_{A \in B} A = U$

ii) Consider U and $R_*(X)$ from B. Let $W = R_*(X)$. Since, $U \cap R_*(X) = R_*(X)$, $W \subset U \cap R_*(X)$ and every X in $U \cap R_*(X)$ belongs to W. If we consider U and $B_R(X)$ from B, taking $W = B_R(X)$, $W \subset U \cap B_R(X)$ and every $X \in U \cap B_R(X)$ belongs to W.

Since $U \cap B_R(X) = B_R(X)$ and when we consider $R_*(X)$ and $B_R(X)$, $R^*(X) \cap B_R(X)$, $R_*(X) \cap B_R(X) = \emptyset$. Thus, B is a basis for τ_R.

Definition 3.5: Let U be the universe and R be an equivalence relation on U. Let τ_R be the rough topology on U and β_R be the basis for τ_R. A subset M of A, the set of attributes is called the **Core** of R if $\beta_M \neq \beta_{R-(r)}$ for every r in M. That is, a core of R is a subset of attributes which is such that none of its elements can be removed without affecting the classification power of attributes.

4. ROUGH TOPOLOGY IN FOOT AND MOUTH DISEASE:

Here we consider the problem of Foot and Mouth Disease, a disease that is a severe, highly contagious viral disease of livestock that has a significant economic impact. The disease affects cattle, swine, sheep, goats and other cloven hoof ruminants. It causes fever, depression, hypersalivation, loss of appetite, weight loss, growth retardation and drop in milk production. In severe cases, it leads to death. The disease is estimated to circulate in **77%** of the global livestock Population, in Africa, the Middle East and Asia, as well as in a limited area of South America. Consider the following information table giving data about 8 patients.

Patients	Loss of appetite	Hypersalivation	Drop in milk production	Fever	Foot and Mouth Disease
P_1	Yes	Yes	Yes	Very high	Yes
P_2	No	Yes	Yes	Normal	No

P_3	No	No	No	Normal	No
P_4	Yes	No	Yes	High	Yes
P_5	Yes	Yes	Yes	Very high	Yes
P_6	Yes	Yes	No	Normal	No
P_7	Yes	No	No	High	No
P_8	Yes	No	No	High	Yes

The columns of the table represent the attributes (the symptoms for Foot and Mouth Disease) and the rows represent the objects (the patients). The entries in the table are the attribute values. The patient P_3 is characterized by the set values (Loss of appetite, No), (Drop in milk production, No), (Hypersalivation, No), (Fever, Normal) and (Food and Mouth Disease, No) which gives information about P_3.

In the table, the patients $P_1, P_4, P_5, P_6, P_7$and P_8 are indiscernible with respect to the attributes "Loss of appetite". The attribute "Loss of appetite" generates two equivalence classes namely, $\{P_1, P_4, P_5, P_6, P_7, P_8\}$ and $\{P_2, P_3\}$, whereas the attributes "Loss of appetite" and "Hypersalivation" generate the equivalence classes $\{P_1, P_5, P_6\}, \{P_2, P_3\}$ and $\{P_4, P_7, P_8\}$.The equivalence classes for the attributes Loss of appetite, Drop in milk production, Hypersalivation and Fever are $\{P_1, P_5\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_6\}$ and $\{P_7, P_8\}$.

For the set of patients Foot and Mouth Disease, lower approximation = $\{P_1, P_4, P_5\}$ and Upper approximation = $\{P_1, P_4, P_5, P_7, P_8\}$and hence the Boundary region = $\{P_7, P_8\}$. Hence, the patients P_7 and P_8 cannot be uniquely classified in view of the available knowledge. The patients P_1, P_4 and P_5 display symptoms which enable us to classify them with certainty as having Foot and Mouth Disease. In our case, the symptoms Loss of appetite, Drop in milk production, Hypersalivation and Fever are considered as **Condition Attributes** and the disease Foot and Mouth Disease is considered as the **Decision Attribute**. Not all condition attributes in an information system are necessary to depict the decision attribute before decision rules are generated. It may happen that the decisionattribute depends not on the whole set of condition attributes but on subset which is given by the core. Here, $U = \{P_1, P_2, P_3 \ldots\ldots P_8\}$.

Case 1: Let $X = \{P_1, P_4, P_5, P_8\}$, the set of patients having foot and mouth disease (FMD). Let R be the equivalence relation on U with respect to the condition attributes. The family of equivalence classes corresponding to R is given by $U/R = \{\{P_1, P_5\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_6\}, \{P_7, P_8\}\}$.The lower and upper approximation of X with respect to R are given by $R_*(X) = \{P_1, P_4, P_5\}$ and$R^*(X) = \{P_1, P_4, P_5, P_7, P_8\}$.Therefore, the rough topology on U with respect to X is given by$\tau_R = \{U, \phi, \{P_1, P_4, P_5,\}, \{P_1, P_4, P_5, P_7, P_8\}, \{P_7, P_8\}\}$. The basis for this topology τ_R is given by $\beta_R = \{U, \{P_1, P_4, P_5\}, \{P_7, P_8\}\}$.

If we remove the attribute 'Loss of appetite' from the set of condition attributes, the family of equivalence classes corresponding to the resulting set of attributes is given by, $U/I(R-(L)) = \{\{P_1, P_5\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_6\}, \{P_7, P_8\}\}$ which is the same as $U/I(R)$ and hence $\tau_{R-(L)} = \tau_R$ and $\beta_{R-(L)} = \beta_R$.When the attribute 'Hypersalivation' is omitted,

$U/I\big(R-(H)\big)=\big\{\{P_1,P_5\},\{P_2\},\{P_3\},\{P_4\},\{P_6,P_7,P_8\}\big\}$,$\big(R-(H)\big)_*(X)=\{P_1,P_4,P_5\}$,$\big(R-(H)\big)^*(X)=\{P_1,P_4,P_5,P_6,P_7,P_8\}$.
Therefore,$\tau_{R-H}=\big\{U,\phi,\{P_1,P_4,P_5\},\{P_1,P_4,P_5,P_6,P_7,P_8\},\{P_6,P_7,P_8\}\big\}$ and its basis $\beta_{R-H}=\big\{U,\{P_1,P_4,P_5\},\{P_6,P_7,P_8\}\big\}\neq\beta_R$.
When the attribute 'Drop in milk production' is omitted$U/I\big(R-(D)\big)=\big\{\{P_1,P_5\},\{P_2\},\{P_3\},\{P_6\},\{P_4,P_7,P_8\}\big\}$,$\big(R-(D)\big)_*(X)=\{P_1,P_5\}$,$(R-D)^*(X)=\{P_1,P_4,P_5,P_7,P_8\}$.Therefore, $\tau_{R-D}=\big\{U,\phi,\{P_1,P_5\},\{P_1,P_4,P_5,P_7,P_8\},\{P_4,P_7,P_8\}\big\}$and itsbasis$\beta_{R-D}=\big\{U,\{P_1,P_5\},\{P_2,P_3,P_7,P_8\}\big\}\neq\beta_R$.On removal of the attribute 'Fever', we get,$U/I\big(R-(F)\big)=\big\{\{P_1,P_5\},\{P_2\},\{P_3\},\{P_4\},\{P_6\},\{P_7,P_8\}\big\}$ which is the same as $U/I(R)$and hence $\tau_{R-F}=\tau_R$ and $\beta_{R-F}=\beta_R$.
If $M=\{H,D\}$,$U/I(r)=\big\{\{P_1,P_2,P_5\},\{P_3,P_7,P_8\},\{P_4\},\{P_6\}\big\}$,$r_*(X)=\{P_4\}$,$r^*(X)=\{P_1,P_2,P_3,P_4,P_5,P_7,P_8\}$,where r is the equivalence relation on U with respect to M.Therefore, the basis for the rough topology is$\beta_M=\big\{U,\{P_4\},\{P_1,P_2,P_3,P_5,P_7,P_8\}\big\}$.Also $\beta_M\neq\beta_{R-(X)}$ for all X in M. Therefore, $CORE(R)=\{H,D\}$

Case 2: Let $=\{P_2,P_3,P_6,P_7\}$, the set of patients not having FMD. Then, $U/I(R)=\big\{\{P_1,P_5\},\{P_2\},\{P_3\},\{P_4\},\{P_6\},\{P_7,P_8\}\big\}$,$R_*(X)=\{P_2,P_3,P_6\}$,$R^*(X)=\{P_2,P_3,P_6,P_7,P_8\}$Therefore,$\tau_R=\big\{U,\phi,\{P_2,P_3,P_6\},\{P_2,P_3,P_6,P_7,P_8\},\{P_7,P_8\}\big\}and\beta_R(X)=\big\{U,\{P_2,P_3,P_6\},\{P_7,P_8\}\big\}$.Omitting the attribute 'Loss of appetite' $U/I\big(R-(L)\big)=\big\{\{P_1,P_5\},\{P_2\},\{P_3\},\{P_4\},\{P_6\},\{P_7,P_8\}\big\}$ which is the same as $U\,/\,I(R)$ and hence $\tau_{R-L}=\tau_R$ and $\beta_{R-(L)}=\beta_R$.If the attribute'Hypersalivation' is removed. $U/I(R-(H))=\big\{\{P_1,P_5\},\{P_2\},\{P_3\}\{P_4\},\{P_6,P_7,P_8\}\big\}$,$\big(R-(D)\big)_*(X)=\{P_2,P_3\}$,$\big(R-(H)\big)^*(X)=\{P_2,P_3,P_6,P_7,P_8\}$.
Therefore,$\tau_{R-(H)}=\big\{U,\phi,\{P_2,P_3\},\{P_2,P_3,P_6,P_7,P_8\},\{P_7,P_8\}\big\}$, $\beta_{R-(D)}=\big\{U,\{P_2,P_3\},\{P_7,P_8\}\big\}\neq\beta_R$. On removal of the attributes 'Drop in milk production', We get, $U/I(R-(D))=\big\{\{P_1,P_5\},\{P_2\},\{P_3\},\{P_4\},\{P_6P_7,P_8\}\big\}$,$\big(R-(D)\big)_*(X)=\{P_2,P_3,P_6\}$,$\big(R-(D)\big)^*(X)=\{P_2,P_3,P_4,P_6,P_7,P_8\}$.
Therefore, $\tau_{R-(D)}=\big\{U,\phi,\{P_2,P_3,P_6\},\{P_2,P_3,P_4,P_6,P_7,P_8\},\{P_4,P_7,P_8\}\big\}$, $\beta_{R-(D)}=\big\{U,\{P_2,P_3,P_6\},\{P_4,P_7,P_8\}\big\}\neq\beta_R$.When the attribute 'Fever' is omitted, $U/I\big(R-(F)\big)=\big\{\{P_1,P_5\},\{P_2\},\{P_3\},\{P_4\},\{P_6\},\{P_7,P_8\}\big\}$which is the same as $U/I(R)$ and hence $\tau_{R-(L)}=\tau_R$ and $\beta_{R-(L)}=\beta_R$.
If $M=\{H,D\}$, $U/I(r)=\big\{\{P_1,P_2,P_5\},\{P_3,P_7,P_8\},\{P_4\},\{P_6\}\big\}$,$r_*(X)=\{P_6\}$,$r^*(X)=\{P_1,P_2,P_3,P_5,P_6,P_7,P_8\}$, where r is the equivalence relation on U with respect to M.Therefore $\beta_M=\big\{U,\{P_3\},\{P_1,P_2,P_3,P_5,P_7,P_8\}\big\}\neq\beta_{R-(X)}$ for every X in M. Therefore, here again, $CORE(R)=\{H,D\}$.

OBSERVATION:

From both cases, we conclude that 'Hypersalivation' and 'Drop in milk production' is the key attributes necessary to decide whether a patienthas Foot and Mouth Disease or not.

The procedure applied in the above two cases can be put in the form of an algorithm as follows:

Algorithm:

Step 1:Given a finite universe U , a finite set A of attributes that is divided into two classes,C of Condition Attributes and D of Decision Attributes, an equivalence relation R on U corresponding to C and a subset X of U, represent the data as an informationtable, columns of which are labelled by attributes, rows by objects and entries of the table are attribute values.
Step 2:Find the Lower Approximation, Upper Approximation and the Boundary Region of X with respect to R.
Step 3:Generate the rough topology $□_{□}$ on □ and its basis $□_{□}$.
Step 4:Remove an attribute x from □ and find the lower and upper approximation and the boundary region of □ with respect to the equivalence on □ − (x).
Step 5:Generate the rough topology $□_{□-(□)}$ on □ and its basis $□_{□-(□)}$
Step 6:Repeat steps 3 and 4 for all attributes in □.
Step7:Those attributes in □ for which $□_{□-(□)} \neq □_{□}$ form the □□□□(□).

5. CONCLUSION:

In this work, we have shown that real world problems can be dealt with the rough topology. The concept of basis has been applied to find the deciding factors of 'Foot and Mouth Disease' which had been reported especially, in Africa. We could find that Drop in milk production and Hypersalivation are the deciding factors for Foot and Mouth Disease. It is also seen that from a clinical point of view, the rough topological model is in par with the medical experts with respect to the diseases analysed here. The proposed rough topology can be applied to more general and complex information systems for future research. The rough set model is based on the original data only and does not need any external information, unlike probability in statistics or grade of membership in the fuzzy set theory. It is also a tool suitable for analysing not only quantitative attributes but also qualitative ones. The results of the rough set model are easy to understand, while the results from other methods need an interpretation of the technical parameters.Thus it is advantageous to use rough topology in real life situations.

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