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Few seperation axioms of $I_{g\delta s}$-closed sets via ideal topological spaces

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Abstract: The main objective of this paper is to introduce the concept of $I_{g\delta s}$-seperation axioms such as $I_{g\delta s}$-T_i spaces where $i = 0, 1, 2$ and investigated their basic properties in ideal topological spaces.

Key Words and Phrases: Ideal topological spaces, ideal seperation axioms, seperation axioms, $I_{g\delta s}$-closed sets.

1. INTRODUCTION

The concept of ideal topology in the classic text was introduced by Kuratowski [9]. D.Jankovie and R Hamlelt [8] introduced the concept of I open set in Ideal Topological Space. After that M.E.Abdel, E.Monsef, F.Iashien and A.A.Nasef [1] introduced a new study about the I open set. The notion of δg–closed sets was first introduced by Dontchev [4] in 1999. Julian Dontchev and maximilian Ganster [5], Yuksel, Acikgoz and Noiri [13] introduced and studied the notions of δ– generalized closed (briefly δg–closed) and δ-closed sets respectively. Ihe concept of seperation axioms in ideal topological spaces was investigated by various authors in [3], [11], [12], [10]. In this paper, we introduce and study the concept of $I_{g\delta s}$-seperation axioms such as $I_{g\delta s}$-T_i spaces where $i = 0,1,2$ with respect to an ideal, and investigated its basic properties.

2. PRELIMINARIES

We start with the definition of closure operator.

Definition 2.1 *[14] An ideal topological space is a topological space (X, τ) with an ideal I on X and it is denoted by (X, τ, I). Given a topological space (X, τ) with an ideal I on X and if $\mathcal{P}(X)$ is the set of all subsets of X, a set operator $(*): \mathcal{P}(X) \to \mathcal{P}(X)$, called a local function of A with respect to τ and I, is defined as follows: for $A \subseteq X$, $A^*(I, \tau) = \{x \in X / U \cap A \notin I \, for \, every \, U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau / x \in U\}$. We simply write $A*$ instead of $A^*(I, \tau)$.*

Definition 2.2 *[14] An ideal I on a set X is a nonempty collection of subsets of X satisfying the following conditions*

1. $A \in I$ and $B \subseteq A$ implies that $B \in I$.
2. $A \in I$ and $B \in I$ implies that $A \cup B \in I$.

Definition 2.3 *[9] Let A be any subset of an ideal topological space (X, τ, I). Define $A^*_{I,\tau} = \{x \in X / U \cap A \notin I \; for \; every \; U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau / x \in U\}$, the collection of all open sets containing x. Let $cl^* = A \cup A^*_{I,\tau}$. Then "$cl^*$" is a Kuratowski closure operator which gives a topology τ^* on X called the $*$-topology.*

Definition 2.4 *[7] Let (X, τ, I) be an ideal space. A subset A of X is said to be*

- I-open if $A \subseteq int(A^*)$.
- semi-I-open if $A \subseteq cl^*(int(A))$.
- pre-I-open if $A \subseteq int(cl^*(A))$.
- α-I-open if $A \subseteq int(cl^*(int(A)))$.
- β-I-open if $A \subseteq cl(int(cl^*(A)))$.

Let (X, τ, I) be an ideal topological space. Then every α-I-open set is α-open, every semi-I-open set is semi-open, every β-I-open set is β-open, every I-open set is pre-I-open, every α-I-open set is pre-I-open, every α-I-open set is semi-I-open, every pre-I-open set is β-I-open; the reverse implication is not true in any of the above.

3. SEPERATION AXIOMS

In this section, we introduce and study weak separation axioms such as $I_{g\delta s}$-T_0, $I_{g\delta s}$-T_1 and $I_{g\delta s}$-T_2 spaces and obtain some of their properties.

Definition 3.1 *A topological space X is said to be $I_{g\delta s}$-T_0 space if for each pair of distinct points x and y of X, there exists a $I_{g\delta s}$-open set containing one point but not the other.*

Theorem 3.2 *A topological space X is a $I_{g\delta s}$-T_0 space if and only if $I_{g\delta s}$-closures of distinct points are distinct.*

Proof Let x and y be distinct points of X. Since X is $I_{g\delta s}$-T_0 space, there exists a $I_{g\delta s}$-open set G such that $x \in G$ and $y \notin G$. Consequently, $X - G$ is a $I_{g\delta s}$-closed set containing y but not x. But $I_{g\delta s}$-$cl\{y\}$ is the intersection of all $I_{g\delta s}$-closed sets containing y. Hence $y \in I_{g\delta s}$-$cl\{y\}$ but $x \notin I_{g\delta s}$-$cl\{y\}$ as $x \notin X - G$. Therefore, $I_{g\delta s}$-$cl\{x\} \neq I_{g\delta s}$-$cl\{y\}$.

Conversely, let $I_{g\delta s}$-$cl\{x\} \neq I_{g\delta s}$-$cl\{y\}$ for $x \neq y$. Then there exists at least one point $z \in X$ such that $z \in I_{g\delta s}$-$cl\{x\}$ but $z \notin I_{g\delta s}$-$cl\{y\}$. We claim $x \notin I_{g\delta s}$-$cl\{y\}$, because if $x \in I_{g\delta s}$-$cl\{y\}$ then $\{x\} \subset I_{g\delta s}$-$cl\{y\}$ implies $I_{g\delta s}$-$cl\{x\} \subset I_{g\delta s}$-$cl\{y\}$. So $z \in I_{g\delta s}$-$cl\{y\}$, which is a contradiction. Hence $x \notin I_{g\delta s}$-$cl\{y\}$, which implies $x \in X - I_{g\delta s}$-$cl\{y\}$, which is a $I_{g\delta s}$-open set containing x but not y. Hence X is $I_{g\delta s}$-T_0 space.

Theorem 3.3 *If $f: X \to V$ is a bijection strongly $I_{g\delta s}$-open and X is $I_{g\delta s}$-T_0 space, then V is also $I_{g\delta s}$-T_0 space.*

Proof Let y_1 and y_2 be two distinct points of V. Since f is bijective there exist distinct points x_1 and x_2 of X such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is $I_{g\delta s}$-T_0 space there exists a $I_{g\delta s}$-open set G such that $x_1 \in G$ and $x_2 \notin G$. Therefore $y_1 = f(x_1) \in f(G)$ and $y_2 = f(x_2) \notin f(G)$. Since f being strongly $I_{g\delta s}$-open function, $f(G)$ is $I_{g\delta s}$-open in V

. Thus, there exists a $I_{g\delta s}$-open set $f(G)$ in V such that $y_1 \in f(G)$ and $y_2 \notin f(G)$. Therefore V is $I_{g\delta s}$-T_0 space.

Definition 3.4 *A topological space X is said to be $I_{g\delta s}$-T_1 space if for any pair of distinct points x and y, there exist a $I_{g\delta s}$-open sets G and H such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.*

Theorem 3.5 *A topological space X is $I_{g\delta s}$-T_1 space if and only if singletons are $I_{g\delta s}$-closed sets.*

Proof Let X be a $I_{g\delta s}$-T_1 space and $x \in K$. Let $y \in K - \{x\}$. Then for $x \neq y$, there exists $I_{g\delta s}$-open set K_y such that $y \in K_y$ and $x \notin K_y$. Consequently, $y \in K_y \subset X - \{x\}$. That is $X - \{x\} = [f K_y : y \in K - \{x\} g$, which is $I_{g\delta s}$-open set. Hence $\{x\}$ is $I_{g\delta s}$-closed set.
Conversely, suppose $\{x\}$ is $I_{g\delta s}$-closed set for every $x \in X$. Let x and $y \in X$ with $x \neq y$. Now $x \neq y$ implies $y \in K - \{x\}$. Hence $K - \{x\}$ is $I_{g\delta s}$-open set containing y but not x. Similarly, $K - \{y\}$ is $I_{g\delta s}$-open set containing x but not y. Therefore X is $I_{g\delta s}$-T_1 space.

Theorem 3.6 *The property being $I_{g\delta s}$-T_1 space is preserved under bijection and strongly $I_{g\delta s}$-open function.*

Proof Let $f: X \to V$ be bijective and strongly $I_{g\delta s}$-open function. Let X be a $I_{g\delta s}$-T_1 space and y_1, y_2 be any two distinct points of V . Since f is bijective there exist distinct points x_1, x_2 of X such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Now X being a $I_{g\delta s}$-T_1 space, there exist $I_{g\delta s}$-open sets G and H such that $x_1 \in G$, $x_2 \notin G$ and $x_1 \notin H$, $x_2 \in H$. Therefore $y_1 = f(x_1) \in f(G)$ but $y_2 = f(x_2) \notin f(G)$ and $y_2 = f(x_2) \in f(H)$ and $y_1 = f(x_1) \notin f(H)$. Now f being strongly $I_{g\delta s}$-open, $f(G)$ and $f(H)$ are $I_{g\delta s}$-open subsets of V such that $y_1 \in f(G)$ but $y_2 \notin f(G)$ and $y_2 \in f(H)$ and $y_1 \notin f(H)$. Hence V is $I_{g\delta s}$-T_1 space.

Theorem 3.7 *Let $f: X \to V$ be bijective and $I_{g\delta s}$-open function. If X is $I_{g\delta s}$-T_1 and $TI_{g\delta s}$-space, then V is $I_{g\delta s}$-T_1 space.*

Proof Let y_1, y_2 be any two distinct points of V. Since f is bijective there exist distinct points x_1, x_2 of X such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Now X being a $I_{g\delta s}$-T_1 space, there exist $I_{g\delta s}$-open sets G and H such that $x_1 \in G$, $x_2 \notin G$ and $x_1 \notin H$, $x_2 \in H$. Therefore $y_1 = f(x_1) \in f(G)$ but $y_2 = f(x_2) \notin f(G)$ and $y_2 = f(x_2) \in f(H)$ and $y_1 = f(x_1) \notin f(H)$. Now X is $I_{g\delta s}$-space which implies G and H are open sets in X and f is $I_{g\delta s}$-open function, $f(G)$ and $f(H)$ are $I_{g\delta s}$-open subsets of V. Thus there exist $I_{g\delta s}$-open sets such that $y_1 \in f(G)$ but $y_2 \notin f(G)$ and $y_2 \in f(H)$ and $y_1 \notin f(H)$. Hence V is $I_{g\delta s}$-T_1 space.

Theorem 3.8 *If $f: X \to V$ is $I_{g\delta s}$-continuous injection and V is T_1 then X is $I_{g\delta s}$-T_1 space.*

Proof Let $f: X \to V$ be $I_{g\delta s}$-continuous injection and V be T_1. For any two distinct points x_1, x_2 of X there exist distinct points y_1, y_2 of V such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since V is T_1 space there exist open sets G and H in V such that $y_1 \in G$, $y_2 \notin G$ and $y_1 \notin H$, $y_2 \in H$. That is $x_1 \in f^{-1}(G)$, $x_1 \notin f^{-1}(H)$ and $x_2 \in f^{-1}(H)$, $x_2 \notin f^{-1}(G)$. Since f is $I_{g\delta s}$-continuous $f^{-1}(G)$, $f^{-1}(H)$ are $I_{g\delta s}$-open sets in X. Thus, for two distinct points x_1,

x_2 of X there exist $I_{g\delta s}$-open sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $x_1 \in f^{-1}(G)$, $x_1 \notin f^{-1}(H)$ and $x_2 \in f^{-1}(H)$, $x_2 \notin f^{-1}(G)$. Therefore X is $I_{g\delta s}$-T_1 space.

Theorem 3.9 *If $f: X \to V$ is $I_{g\delta s}$-irresolute injective function and V is $I_{g\delta s}$-T_1 space then X is $I_{g\delta s}$-T_1 space.*

Proof Let x_1, x_2 be pair of distinct points in X. Since f is injective there exist distinct points y_1, y_2 of V such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since V is $I_{g\delta s}$-T_1 space there exist $I_{g\delta s}$-open sets G and Y in V such that $y_1 \in G$, $y_2 \notin G$ and $y_1 \notin H$, $y_2 \in H$. That is $x_1 \in f^{-1}(G)$, $x_1 \notin f^{-1}(H)$ and $x_2 \in f^{-1}(H)$, $x_2 \notin f^{-1}(G)$. Since f is $I_{g\delta s}$-irresolute $f^{-1}(G)$, $f^{-1}(H)$ are $I_{g\delta s}$-open sets in X. Thus, for two distinct points x_1, x_2 of X there exist $I_{g\delta s}$-open sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $x_1 \in f^{-1}(G)$, $x_1 \notin f^{-1}(H)$ and $x_2 \in f^{-1}(H)$, $x_2 \notin f^{-1}(G)$. Therefore X is $I_{g\delta s}$-T_1 space.

Definition 3.10 *A topological space X is said to be $I_{g\delta s}$-T_2 space if for any pair of distinct points x and y, there exist disjoint $I_{g\delta s}$-open sets G and H such that $x \in G$ and $y \in H$.*

Theorem 3.11 *If $f: X \to V$ is $I_{g\delta s}$-continuous injection and V is T_2 then X is $I_{g\delta s}$-T_2 space.*

Proof Let $f: X \to V$ be $I_{g\delta s}$-continuous injection and V be T_2. For any two distinct points x_1, x_2 of X there exist distinct points y_1, y_2 of V such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since V is T_2 space there exist disjoint open sets G and H in V such that $y_1 \in G$ and $y_2 \in H$. That is $x_1 \in f^{-1}(G)$ and $x_2 \in f^{-1}(H)$. Since f is $I_{g\delta s}$-continuous $f^{-1}(G)$, $f^{-1}(H)$ are $I_{g\delta s}$-open sets in X. Further f is injective, $f^{-1}(G) f^{-1}(H) = f^{-1}(G \ H) = f^{-1}(\emptyset) = \emptyset$. Thus, for two disjoint points x_1, x_2 of X there exist disjoint $I_{g\delta s}$-open sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $x_1 \in f^{-1}(G)$ and $x_2 \in f^{-1}(H)$. Therefore X is $I_{g\delta s}$-T_2 space.

Theorem 3.12 *If $f: X \to V$ is $I_{g\delta s}$-irresolute injective function and V is $I_{g\delta s}$-T_2 space then X is $I_{g\delta s}$-T_2 space.*

Proof Let x_1, x_2 be pair of distinct points in X. Since f is injective there exist distinct points y_1, y_2 of V such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since V is $I_{g\delta s}$-T_2 space there exist disjoint $I_{g\delta s}$-open sets G and H in V such that $y_1 \in G$ and $y_2 \in H$. That is $x_1 \in f^{-1}(G)$ and $x_2 \in f^{-1}(H)$. Since f is $I_{g\delta s}$-irresolute injective $f^{-1}(G)$, $f^{-1}(H)$ are distinct $I_{g\delta s}$-open sets in X. Thus, for two disjoint points x_1, x_2 of X there exist disjoint $I_{g\delta s}$-open sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $x_1 \in f^{-1}(G)$ and $x_2 \in f^{-1}(H)$. Therefore X is $I_{g\delta s}$-T_2 space.

Definition 3.13 *A topological space X is called δ-$T_{3/4}$ if every $I_{g\delta s}$-closed set in it is δ-closed.*

Theorem 3.14 *For a topological space X, if X is a δ-$T_{3/4}$ space, then every singleton set $\{x\}$ is δ-open or δ-closed.*

Proof Suppose X is a δ-$T_{3/4}$ space. If $\{x\}$ is not δ-closed, then $X - \{x\}$ is not δ-open. Then the only δ-open set containing $X - \{x\}$ is X. Therefore $X - \{x\}$ is $I_{g\delta s}$-closed set of X. Since X is a δ-$T_{3/4}$ space, $X - \{x\}$ is δ-closed, which implies $\{x\}$ is δ-open.

Theorem 3.15 *Every δ-$T_{3/4}$ space is $T_{3/4}$ space.*

Proof Follows from the fact that every δ-g-closed set is $I_{g\delta s}$-closed set.

Definition 3.16 *A subset A of X is δ-nowhere dense if $int(\delta\text{-}I\text{-}cl(A)) = \emptyset$.*

Lemma 3.17 *For a topological space X the following are valid*

1. Every singleton set is δ-pre closed or δ-open in X.
2. Every singleton set is δ-nowhere dense or δ-pre open in X.

Theorem 3.18 *For a topological space the following are equivalent*

1. X is δ-$T_{3/4}$ space.
2. Every δ-pre closed singleton set of X is δ-closed.
3. Every non δ-open singleton set of X is δ-closed.

Proof$(i) \Rightarrow (ii)$ Let $x \in X$ and $\{x\}$ be δ-pre closed in X. By above lemma, $\{x\}$ is not δ-open and hence by theorem 2.3.2, $\{x\}$ is δ-closed.
$(ii) \Rightarrow (i)$ If $\{x\}$ is not δ-open for some $x \in X$, then by lemma 2.3.5, it is δ-pre closed and by (ii) it is δ-closed. Hence X is δ-$T_{3/4}$ space.
$(ii) \Rightarrow (iii)$ If $\{x\}$ is not δ-open for some $x \in X$, by lemma 2.3.5, $\{x\}$ is δ- pre closed and by (ii), it is δ-closed.
$(iii) \Rightarrow (ii)$ Let $\{x\}$ be δ-pre closed in $\{x\}$. By lemma 2.3.5, $\{x\}$ is not δ-open and hence by (iii), it is δ-closed.

Definition 3.19 *A topological space X is called δ-$T_{1/2}$ space if every $I_{g\delta s}$-closed set in it is semi closed.*

Theorem 3.20 *For a topological space X the following conditions are equivalent*

1. X is δ-$T_{1/2}$ space.
2. Every singleton set is either δ-closed or semi open.

Proof$(i) \Rightarrow (ii)$ If $\{x\}$ is not δ-closed, then $X - \{x\}$ is not δ-open. Then the only δ-open set containing $X - \{x\}$ is X. Therefore $X - \{x\}$ is $I_{g\delta s}$-closed set in X. By (i), $X - \{x\}$ is semi closed, which implies $\{x\}$ is semi open.
$(ii) \Rightarrow (i)$ Let $A \subseteq X$ be $I_{g\delta s}$-closed set and $x \in I_s cl(A)$. Then consider the following cases
Case (1): Let $\{x\}$ be δ-open. Since $x \in I_s cl(A)$, then $\{x\} \cap I_s cl(A) \neq \emptyset$. This implies $x \in A$.
Case (2): Let $\{x\}$ be δ-closed. Assume that $x \notin A$, then $x \in I_s cl(A) - A$, which implies $\{x\} \subset I_s cl(A) - A$. This is not possible according to theorem 2.2.9. This shows that, $x \in A$.
So in both cases, $I_s cl(A) \subset A$. Since the reverse inclusion is trivial, implies $I_s cl(A) = A$. Therefore A is semi closed.

Theorem 3.21 *Every δ-$T_{3/4}$ space is $I_{g\delta s}$-$T_{1/2}$ space.*

Proof Let X be a ,δ-$T_{3/4}$ space. Then by theorem 2.3.2, every singleton set of X is δ-open or δ-closed. But every δ-open set is semi open set. Thus every singleton set of X is semi open or δ-closed. By theorem 2.3.8, X is $I_{g\delta s}$-$T_{1/2}$ space.

Theorem 3.22 *Every $I_{g\delta s}$-$T_{1/2}$ space is semi-$T_{1/2}$ space.*

Proof Let A be sg-closed subset of X. Since every sg-closed set is $I_{g\delta s}$-closed and X is $I_{g\delta s}$-$T_{1/2}$ space, implies A is semi closed. Hence X is semi-$T_{1/2}$.

Theorem 3.23 *For any topological space X*
1. $I_sO(X) \subset I_{g\delta s}O(X)$.
2. A space X is δ-$T_{1/2}$ space if and only if $I_sO(X) = I_{g\delta s}O(X)$.

Proof (i) if X is semi open, then $X - A$ is semi closed. So $X - A$ is $I_{g\delta s}$-closed, this implies A is $I_{g\delta s}$-open. Hence $I_sO(X) \subset I_{g\delta s}O(X)$.
(ii) Let A be a $I_{g\delta s}$-$T_{1/2}$ space and $A \in I_{g\delta s}O(X)$. Then $X - A$ is $I_{g\delta s}$-closed set. By hypothesis, $X - A$ is semi closed and hence $A \in I_sO(X)$. Therefore $I_{g\delta s}O(X) \subset I_sO(X)$. By (i), $I_sO(X)C \subset I_{g\delta s}O(X)$. Therefore $I_sO(X) = I_{g\delta s}O(X)$.
Conversely, let $I_sO(X) = I_{g\delta s}O(X)$ and A be a $I_{g\delta s}$-closed set. Then $X - A$ is $I_{g\delta s}$-open. Hence $X - A$ is semi open, which implies A is semi closed. Thus every $I_{g\delta s}$-closed set is semi closed. Therefore X is $I_{g\delta s}$-$T_{1/2}$-space.

Lemma 3.24 *For a space X the following are equivalent*
1. Every δ-pre open singleton set is δ-closed.
2. Every singleton set is δ-nowhere dense or δ-closed.

Proof$(i) \Rightarrow (ii)$ By Lemma 2.3.5, every singleton set is either δ-nowhere dense or δ-pre open. In the first case we are done and in the second case δ-closedness follows from the assumption.
$(ii) \Rightarrow (i)$ Let $\{x\}$ be δ-pre open. Assume that $\{x\}$ is not δ-closed. Then by (ii), it is δ-nowhere dense. Thus $\{x\} \subset int(\delta\text{-}I\text{-}\ cl\{x\}) = \emptyset$, which is not possible. Hence $\{x\}$ is δ-closed.

Theorem 3.25 *For a space X the following are equivalent*
1. X is δ-T_1 space.
2. X is δ-$T_{3/4}$ space and every singleton set is δ-nowhere dense or δ-closed.
3. X is δ-$T_{3/4}$ space and every δ-pre open singleton set is δ-closed.

Proof$(i) \Rightarrow (ii)$ Obvious.
$(ii) \Rightarrow (i)$ If singleton set is not δ-closed, then it must be δ-open, since X is δ-$T_{3/4}$-space. But singleton set is δ-open if and only if it is regular open, which implies singleton set is regular open. Moreover, by rest of assumption X is δ-nowhere dense at the same time, X must be δ-T_1-space.
$(ii) \Rightarrow (iii)$ Follows from lemma 2.3.15.

Theorem 3.26 *In any topological space the following are equivalent*
1. X is $I_{g\delta s}$-T_2 space.
2. For each $x \neq y$, there exists a $I_{g\delta s}$-open set G such that $x \in G$ and $y \notin I_{g\delta s}$-$cl(G)$.
3. For each $x \in G$, $\{x\} = \cap \{I_{g\delta s}\text{-}cl(G): G$ is a $I_{g\delta s}$-open set in X and $x \in G\}$.

Proof $(1) \Rightarrow (2)$: Assume (1) holds. Let $x \in G$ and $x \neq y$, then there exist disjoint $I_{g\delta s}$-open sets G and H such that $x \in G$ and $Y \in V$. Clearly, $X - Y$ is $I_{g\delta s}$-closed set. Since $G \cap Y = \emptyset$, $G \subset X - Y$. Therefore $I_{g\delta s}$-$cl(G) \subset I_{g\delta s}$-$cl(X - Y) = X - Y$. Now $y \notin X - Y$ implies $y \notin I_{g\delta s}$-$cl(G)$.
$(2) \Rightarrow (3)$: For each $x \neq y$, there exists a $I_{g\delta s}$-open set G such that $x \in G$ and $y \notin I_{g\delta s}$-$cl(G)$. So $y \notin I_{g\delta s}$-$cl(G)$: G is a $I_{g\delta s}$-open set in X and $x \in G\} = \{x\}$.
$(3) \Rightarrow (1)$: Let $x, y \in X$ and $x \neq y$. By hpothesis there exists a $I_{g\delta s}$-open set G such that $x \in G$ and $y \notin I_{g\delta s}$-$cl(G)$. This implies there exists a $I_{g\delta s}$-closed set H such that $y \notin H$. Therefore $y \in X - H$ and $X - H$ is $I_{g\delta s}$-open set. Thus, there exist two disjoint $I_{g\delta s}$-open sets G and $X - H$ such that $x \in G$ and $y \in X - H$. Therefore X is $I_{g\delta s}$-T_2 space.

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