International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021

# Decomposition Of Various Graphs In To Subtractdivisor Cordial Graphs

N Vinothkumar[1], NPreethi[2], K Nivethitha[3]

[1] Department of Mathematics, Bannari Amman Institute of Technology, Erode, Tamilnadu.
[2]Department of Science and Humanities, Sri Krishna College of Engineering and Technology, Coimbatore, Tamilnadu.
[3]Department of Mathematics, M Kumarasamycollege of Engineering, Karur, Tamilnadu.
India
Email: vinoth@bitsathy.ac.in[1], preethin@skcet.ac.in[2],
knivethitha13@gmail.com[3]

***Abstract: A subtract divisor cordial labeling of a graph G with vertex set V is a bijection*** $f: V \to \{1,2,3,...|V(G)|\}$ ***and the edge labeling*** $f^*: E \to \{0,1\}$ ***is defined by*** $f^*(uv) = 1$***, if 2 divides*** $f(u) - f(v)$ ***and 0 otherwise. The function f is called a subtractdivisor cordial labeling if*** $\left|e_{f^*}(0) - e_{f^*}(1)\right| \le 1$***.That is the number of edges labeled with 0 and the number of edges labeled with 1 differs by at most 1. A graph with a subtract divisor cordial labeling is called a subtract divisor cordial graph.A decomposition of*** $G$ ***is a collection*** $\psi_S = \{H_1, H_2, .....H_r\}$ ***such that*** $H_i$ ***are edge disjoint and every edges in*** $H_i$ ***belongs to*** $G$***. If each*** $H_i$ ***is a subtract divisor cordial graphs, then*** $\psi_S$ ***is called a subtract divisor cordial decompositionof*** $G$***. The minimum cardinality of a subtract divisor cordial decomposition of*** $G$ ***is called the subtract divisor cordial decomposition number of*** $G$ ***and it is denoted by*** $\pi_S(G)$***. In this paper we define subtractdivisor cordialdecomposition and subtract divisor cordial decomposition number*** $\pi_S(G)$ ***of a graphs. Also investigate some bounds of*** $\pi_{SUB}(G)$ ***in product graphs like Cartesian product, composition etc.***

***Keywords: Subtract divisor cordial, subtract divisor cordial decomposition and subtract divisor cordial decomposition number.***

## 1. INTRODUCTION

For all further usual terms and notations we followHarary [1]. A labeling of a graph is a mapping that transfers the graph components to the set of numbers, typically to the set of non-negative or positive integers. If the domain is the set of vertices the labeling is called vertex labeling. If the domain is the set of edges then the labeling is called edge labeling. If the labels are allocated to both vertices and edges then the labeling is called total labeling. A. Lourdusamy and F. Patrick introduced the concept of subtractdivisor cordial labeling in [2,3].

A graph is an well-ordered pair $G = (V, E)$, where $V$ is a non-empty finite set, called the set of vertices or nodes of G, and $E$ is a set of unordered pairs (2-element subsets) of $V$, called the edges of $G$ . If $xy \in E$ , x and y are called adjacent and they are incident with the edge $xy$ .

The complete graph on n vertices, denoted by $K_n$ , is a graph on n vertices such that every pair of vertices is connected by an edge. The empty graph on n vertices, denoted by $E_n$ , is a graph on n vertices with no edges.A graph $G' = (V', E')$ is a sub graph of $G = (V, E)$ if and only if $V' \subseteq V$ and $E' \subseteq E$ .The order of a graph $G = (V, E)$ is $|V|$, the number of its vertices. The size of G is $|E|$, the quantity of its edges. The degree of a node $x \in V$ , represented by $d(x)$, is the quantity of edges incident with it.

A subgraph H of G is a graph such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ . For a graph $G(V, E)$ and a subset $W \subseteq V$ , the subgraph of G induced by W, denoted as $G[W]$, is the graph $H(W, F)$ such that, for all $u, v \in W$ , if $uv \in E$ , then $uv \in F$ . We say H is an induced subgraph of G.

A graph $G(V, E)$ is said to be connected if every pair of vertices is connected by a path. If there is exactly one path connecting each pair of vertices, we say G is a tree. Equivalently, a tree is a connected graph with n − 1 edges. A pathgraph $P_n$ is a connected graph on n vertices such that each vertex has degree at most 2. A cycle graph $C_n$ is a connected graph on n vertices such that every vertex has degree 2.

A complete graph $P_n$ is a graph with n vertices such that every vertex is adjacent to all the others. On the other hand, an independent set is a set of vertices of a graph in which no two vertices are adjacent. We denote In for an independent set with n vertices.

A bipartite graph $G(V, E)$ is a graph such that there exists a partition $P(A, B)$ of V such that every edge of G connects a vertex in A to one in B. Equivalently, G is said to be bipartite if A and B are independent sets. The bipartite graph is also denoted as $G(A, B, E)$

The Brush graph $B_n$ , $(n \geq 2)$ can be constructed by path graph $P_n$ , $(n \geq 2)$ by joining the star graph $K_{1,1}$ at each vertex of the path. i.e., $B_n = P_n + nK_{1,1}$ .

In this paper we define subtract divisor cordialdecomposition and subtract divisor cordial decomposition number $\pi_S(G)$ of a graphs. Also investigate some bounds of $\pi_{SUB}(G)$ in product graphs like Cartesian product, composition etc.

## 2. SUBTRACTDIVISOR CORDIAL DECOMPOSITION

In this section we define subtractdivisor cordialdecomposition of a graph $G(V, E)$ and investigate some bounds of subtractdivisor cordialdecomposition number in various graphs $G(V, E)$ .

**Definition 2.1.:** A subtract divisor cordial labeling of a graph G with vertex set V is a bijection $f : V \to \{1,2,3,...|V(G)|\}$ and the edge labeling $f^* : E \to \{0,1\}$ is defined by $f^*(uv) = 1$ , if 2 divides $f(u) + f(v)$ and 0 otherwise. The function f is called a subtractdivisor cordial labeling if $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$.That is the number of edges labeled with 0 and the number of edges labeled with 1 differs by at most 1.

A graph with a subtract divisor cordial labeling is called a subtract divisor cordial graph.

**Definition 2.2:** A decomposition of $G$ is a collection $\psi_S = \{H_1, H_2, .....H_r\}$ such that $H_i$ are edge disjoint and every edges in $H_i$ belongs to $G$. If each $H_i$ is a subtractdivisor cordial graphs, then $\psi_{SUB}$ is called a subtract divisor cordial decompositionof $G$. The minimum cardinality of a subtractdivisor cordial decomposition of $G$ is called the subtract divisor cordial decomposition number of $G$ and it is denoted by $\pi_S(G)$.

**Definition 2.3:** Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. The join $G_1 + G_2$ of $G_1$ *and* $G_2$ with disjoint vertex set $V_1 \& V_2$ and the edge set E of $G_1 + G_2$ is defined by the two vertices $(u_i, v_j)$ if one of the following conditions are satisfied

i) $u_i v_j \in E_1$.

ii) $u_i v_j \in E_2$.

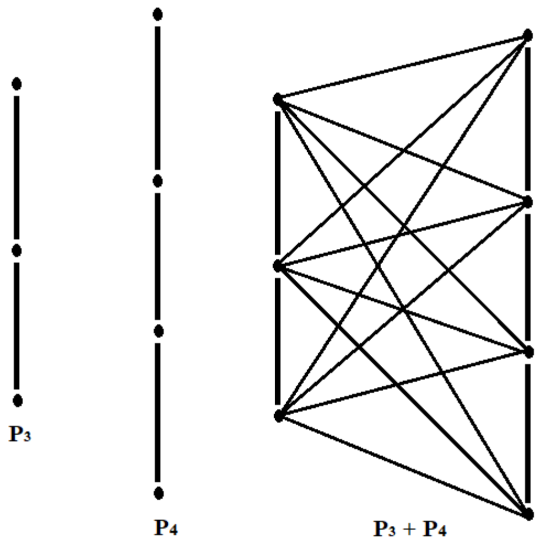
iii) $u_i \in V_1 \; \& v_j \in V_2$ , $u_i v_j \in E$

**Theorem 2.1:** A graph $(P_m + P_n)$ is a join of two path subtract divisor cordial graphs with $(m < n)$ .The bounds of subtract divisor cordial decomposition number of the graph $(P_m + P_n)$ is, $3 \leq \pi_{SUB}(P_m + P_n)$..

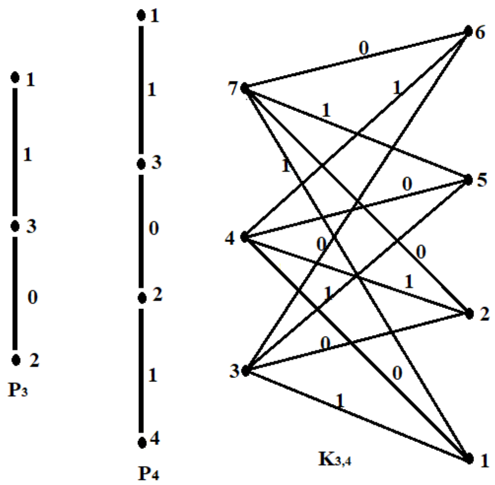
**Proof:**Let $P_m$ and $P_n$ be two pathsubtract divisor cordialgraphs of order m and n $(m > n)$ respectively and $(P_m + P_n)$ is a join of $P_m$ and $P_n$ with edge set E. The graph $(P_m + P_n)$ contains $(m+n)$ vertices and the edge set is $E = E_1 \cup E_2 \cup S(K_{m,n})$, Here $S(K_{m,n})$ is a size of a complete bipartite graph $K_{m,n}$.In the graph $(P_m + P_n)$ there are graphs $P_m$ , $P_n$ and the complete bipartite graphs $K_{m,n}$ .Note that $P_m$ and $P_n$ be two subtract divisor cordialgraphs and complete bipartite graphs $K_{m,n}$ also subtract divisor cordialgraph. This implies $\psi_S \supseteq \{P_m \cup P_n \cup K_{mn}\}$ and $|\psi_S| \geq |\{P_m \cup P_n \cup K_{mn}\}|$. Note that the graphs $P_m$ , $P_n$ and $K_{m,n}$ are subtract divisor cordialgraphs. Hence $\pi_{SUB}(P_m + P_n) \geq (3)$.

**Illustration 2.1:** The Join of two subtract divisor cordial graphs $P_2 \& P_3$ is given in figure.2.1

International Journal of Aquatic Science
ISSN: 2008-8019
Vol 12, Issue 02, 2021



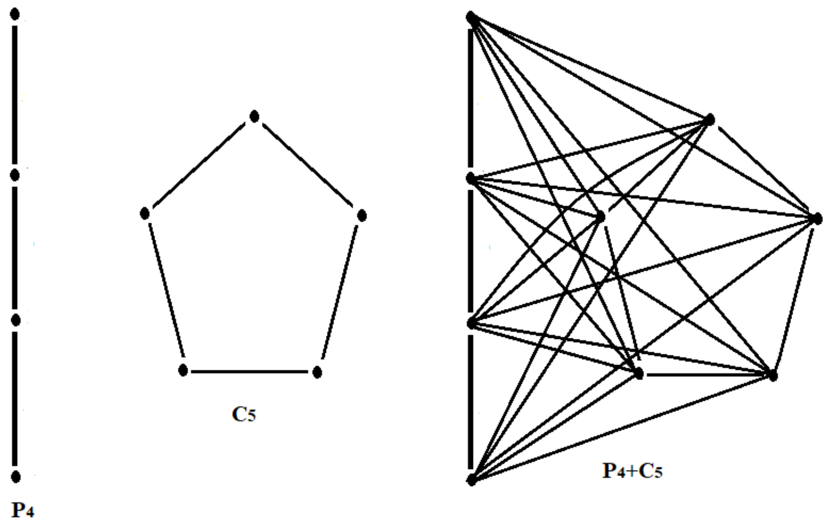

Decomposition of the graph $(P_2 + P_3)$ in to minimum copies of subtract divisor cordial graph.This implies the lower bound of $\pi_{SUB}(P_3 + P_4)$ is $3 \le \pi_S(P_2 + P_3)$.



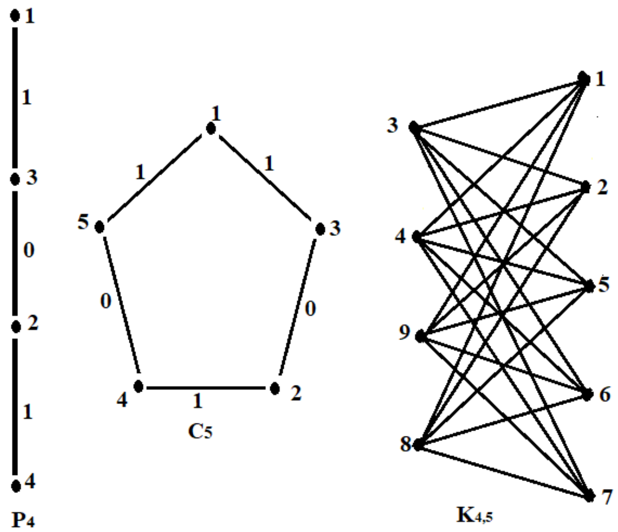

**Theorem 2.2.**A graph $(P_m + C_n)$ is a join of subtract divisor cordialgraphs $P_m$ and cycle $C_n$ with $(m < n)$. The bounds of subtract divisor cordialdecomposition numberof the graph $(P_m + C_n)$ is, $3 \le \pi_{SUB}(P_m + C_n)$..

**Proof:**Let $P_m$ and $C_n$ be two pathsubtract divisor cordialgraphs of order m and n $(m > n)$ respectively and $(P_m + C_n)$ is a join of $P_m$ and $C_n$ with edge set E. The graph $(P_m + C_n)$ contains $(m + n)$ vertices and the edge set is $E = E_1 \cup E_2 \cup S(K_{m,n})$, Here $S(K_{m,n})$ is a size of a complete bipartite graph $K_{m,n}$.In the graph $(P_m + C_n)$ there are graphs $P_m$ , $P_n$ and the complete bipartite graphs $K_{m,n}$ .Note that $P_m$ and $C_n$ be two subtract divisor cordialgraphs and complete bipartite graphs $K_{m,n}$ also subtract divisor cordialgraph. This implies $\psi_p \supseteq \{P_m \cup C_n \cup K_{mn}\}$ and $|\psi_p| \geq |\{P_m \cup C_n \cup K_{mn}\}|$. Note that the graphs $P_m$ , $C_n$ and $K_{m,n}$ are subtractdivisor cordialgraphs. Hence $\pi_{SUB}(P_m + C_n) \geq (3)$.

**Illustration 2.2:** The Join of two subtract divisor cordial graphs $P_2 \& P_3$ is given in figure.2.2




Decomposition of the graph $(P_4 + C_5)$ in to minimum copies of subtract divisor cordial graph this implies the bound of $\pi_{SUB}(P_4 + C_5)$ is $3 \leq \pi_{SUB}(P_4 + C_5)$




**Definition 2.4:** Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. The Cartesian product $G_1 \times G_2$ of $G_1$ *and* $G_2$, is a graph with vertex set $V = V_1 \times V_2$ and the edge set of $G_1 \times G_2$ is defined by the two vertices $(u_i, v_j)$ & $(u_k, v_l)$ if one of the following conditions are satisfied

i) $u_1 = v_1$ and $u_2, v_2$ are adjacent vertices in $G_2 = (V_2, E_2)$.
ii) $u_2 = v_2$ and $u_1, v_1$ are adjacent vertices in $G_1 = (V_1, E_1)$.

**Theorem 2.3:** A graph $(P_m \times P_n)$ is a Cartesian product of two subtractdivisor cordialgraphs $(P_m \times P_n)$ with order m and n. Then bounds of subtractdivisor cordial decomposition numberof the graph $(P_m \times P_n)$ is, $m+n \le \pi_S(P_m \times P_n)$.

**Proof:** Let $P_m$ and $P_n$ be two pathsubtractdivisor cordial graphs of order m and n respectively and $(P_m \times P_n)$ is a Cartesian product of $P_n$ & $P_m$ with edge set E. An edge $((x_1x_2)(y_1y_2)) \in E$ satisfies one of the following conditions

i) $x_1 = y_1$ and $x_2, y_2$ are adjacent vertices in $G_2 = (V_2, E_2)$.
ii) $x_2 = y_2$ and $x_1, y_1$ are adjacent vertices in $G_1 = (V_1, E_1)$.

**Case (i):** If $x_1 = y_1$ and $x_2, y_2$ are adjacent vertices in $G_2 = (V_2, E_2)$

If $x_1 = y_1$ and $x_2, y_2$ are adjacent vertices in $P_n$ . Let the sub graph $H_i$ is isomorphic to the graph $P_n$ . In $(P_m \times P_n)$ there are 'm' copies of graph $P_n$ and it is subtractdivisor cordial graph .This implies $H_i$ is also asubtractdivisor cordial graph. This implies $H_i \subset \psi$

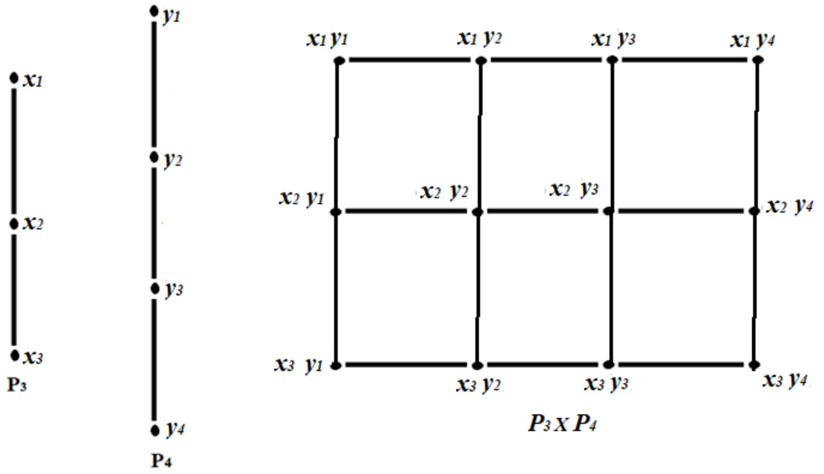
**Case (ii):** If $x_2 = y_2$ and $x_1, y_1$ are adjacent vertices in $P_m$

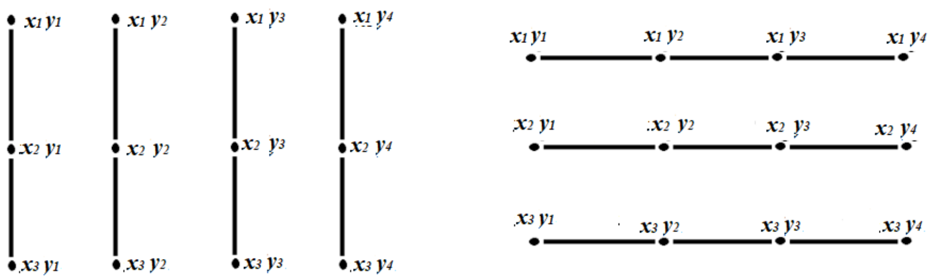
If $x_2 = y_2$ and $x_1, y_1$ are adjacent vertices in $P_m$ . Note that the sub graph $H_j$ is isomorphic to the graph $P_m$ . In $(P_m \times P_n)$ there are 'n' copies of graph $P_m$ and it is subtractdivisor cordial graph This implies $H_j$ is also asubtractdivisor cordial graph. This implies $H_j \subset \psi$.

From case (i) and (ii), we get $\psi = \left\{\left(\bigcup_{i=1}^{m} H_i\right) \cup \left(\bigcup_{j=1}^{n} H_j\right)\right\}$ this implies $|\psi| = \sum_{i=1}^{m} H_i + \sum_{j=1}^{n} H_j = m+n$.

In $(P_m \times P_n)$ there are $n(m-1)+m(n-1) \Rightarrow 2(nm)-(m+n)$ edges. Note that every edge is a subtractdivisor cordial graph. This implies $\pi_p(P_m + P_n) \le 2(nm-1)$. Hence we get $m+n \le \pi_p(P_m + P_n) \le 2(nm)-(m+n)$.

**Illustration 2.3:** The Cartesian product of two subtractdivisor cordial $P_3$ & $P_4$ is given in Figure.2.3




4 copies of $P_3$

3 copies of $P_3$

Note that the lower bound of $\pi_S(P_3+P_4)$ is $7 \le \pi_S(P_3+P_4)$.

**Theorem 2.4:** A graph $(P_m \times C_n)$ is a Cartesian product of two subtractdivisor cordialgraphs $(P_m \times C_n)$ with order m and n. Then bounds of subtractdivisor cordialdecomposition numberof the graph $(P_m \times C_n)$ is, $m+n \le \pi_p(P_m \times C_n) \le 2(nm-1)$.

**Proof:**Let $P_m$ and $C_n$ be two pathsubtractdivisor cordialgraphs of order m and n respectively and $(P_m \times C_n)$is a Cartesian product of $P_m$ & $C_n$with edge set E. An edge $((x_1x_2)(y_1y_2)) \in E$ satisfies one of the following conditions

i) $x_1 = y_1$ and $x_2, y_2$ are adjacent vertices in $G_2 = (V_2, E_2)$.

ii) $x_2 = y_2$ and $x_1, y_1$ are adjacent vertices in $G_1 = (V_1, E_1)$.

**Case (i):**If $x_1 = y_1$ and $x_2, y_2$ are adjacent vertices in $C_n$.

If $x_1 = y_1$ and $x_2, y_2$ are adjacent vertices in $C_n$ . Let the sub graph $H_i$ is isomorphic to the graph $C_n$. In $(P_m \times C_n)$ there are 'm' copies of graph $C_n$ and it is subtractdivisor cordialgraph .This implies $H_i$ is also subtract divisor cordialgraph. This implies $H_i \subset \psi$

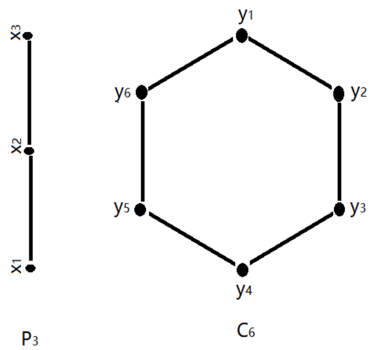
**Case (ii):**If $x_2 = y_2$ and $x_1, y_1$ are adjacent vertices in $P_m$

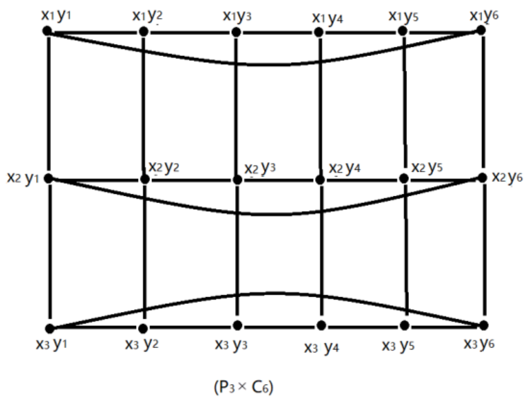
If $x_2 = y_2$ and $x_1, y_1$ are adjacent vertices in $P_m$ . Note that the sub graph $H_j$ is isomorphic to the graph $P_m$ . In $(P_m \times C_n)$ there are 'n' copies of graph $P_m$ and it is subtractdivisor cordialgraph This implies $H_j$ is also asubtractdivisor cordialgraph. This implies $H_j \subset \psi$ .

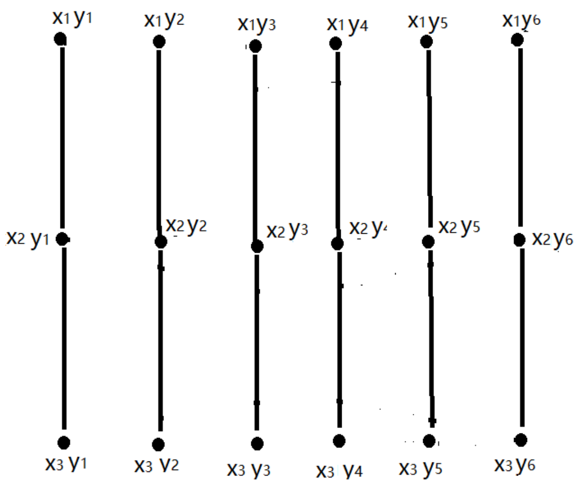
From case (i) and (ii), we get $\psi = \left\{ \left( \bigcup_{i=1}^{m} H_i \right) \cup \left( \bigcup_{j=1}^{n} H_j \right) \right\}$ this implies $|\psi| = \sum_{i=1}^{m} H_i + \sum_{j=1}^{n} H_j = m + n$.

In $(P_m \times C_n)$there are $n(m-1) + m(n) \Rightarrow 2(nm) - 1$edges. Note that every edge is a subtractdivisor cordialgraph. This implies $\pi_p(P_m \times C_n) \le 2(nm) - 1$. Hence we get $m + n \le \pi_p(P_m \times C_n) \le 2(nm) - 1$.

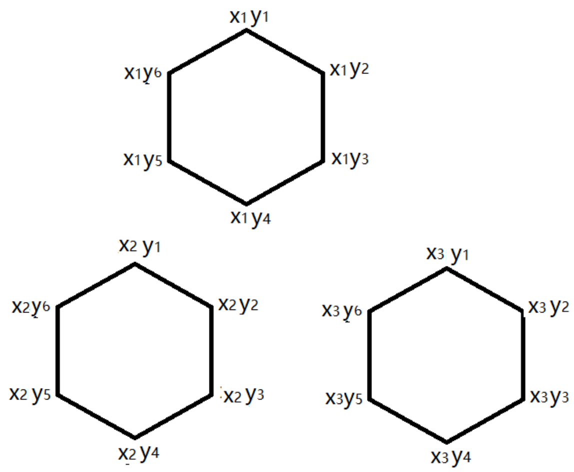
**Illustration 2.4:** The Cartesian product of two subtractdivisor cordialgraphs $P_3 \& C_6$ is given in Figure.2.3.The bound of $\pi_p(P_3 \times C_6)$ is $9 \le \pi_p(P_3 \times C_6) \le 35$.




6 copies of $P_3$




3 copies of $C_6$

**Definition 2.4:** Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. The Composition $G_1 \circ G_2$ of $G_1$ *and* $G_2$, is a graph with vertex set $V = V_1 \times V_2$ and the edges in $G_1 \circ G_2$ is defined by the two vertices $(u_1, u_2)$ & $(v_1, v_2)$ if one of the following conditions are satisfied

i) $u_1 = v_1$ and $u_2, v_2$ are adjacent vertices in $G_2 = (V_2, E_2)$.
ii) $u_2 = v_2$ and $u_1, v_1$ are adjacent vertices in $G_1 = (V_1, E_1)$.
iii) $u_1, v_1$ are adjacent vertices in $G_1 = (V_1, E_1)$.

**Theorem 2.5:** A graph $G_1 \circ G_2$ is a Composition of two subtractdivisor cordialgraphs $G_1 \& G_2$ with order m and n, can be decomposed in to at least $(mn + m + n)$ subtractdivisor cordialgraphs (i.e $\pi_{SUB}(G_1 \circ G_2) \geq (mn + m + n)$).

**Proof:**Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two subtractdivisor cordialgraphs of order m and n respectively and $G_1 \circ G_2$ is a Composition of $G_1$ *and* $G_2$ with edge set E the one of the following conditions are satisfied

i) $u_1 = v_1$ and $u_2, v_2$ are adjacent vertices in $G_2 = (V_2, E_2)$.
ii) $u_2 = v_2$ and $u_1, v_1$ are adjacent vertices in $G_1 = (V_1, E_1)$.
iii) $u_1, v_1$ are adjacent vertices in $G_1 = (V_1, E_1)$.

**Case (i):**If $u_1 = v_1$ and $u_2, v_2$ are adjacent vertices in $G_2 = (V_2, E_2)$

If $u_1 = v_1$ and $u_2, v_2$ are adjacent vertices in $G_2 = (V_2, E_2)$ . Let the sub graph $H_i$ is isomorphic to the graph $G_2 = (V_2, E_2)$. The graph $G_2 = (V_2, E_2)$ be a subtractdivisor cordialgraph this implies $H_i$ is also asubtractdivisor cordialgraph. This implies $H_i \subset \psi$

**Case (ii):**If $u_2 = v_2$ $u_1, v_1$ are adjacent vertices in $G_1 = (V_1, E_1)$

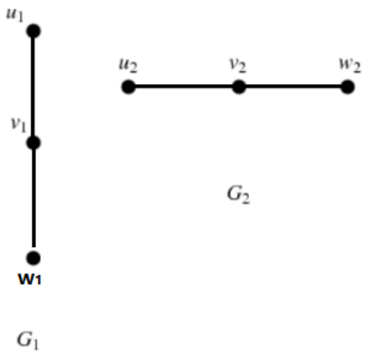
If $u_2 = v_2$ $u_1, v_1$ are adjacent vertices in $G_1 = (V_1, E_1)$ . Let the sub graph $H_j$ is isomorphic to the graph $G_1 = (V_1, E_1)$. The graph $G_1 = (V_1, E_1)$ be a subtractdivisor cordialgraph this implies $H_j$ is also asubtractdivisor cordialgraph. This implies $H_j \subset \psi$ .

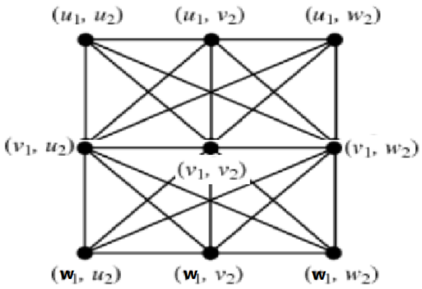
**Case (iii):** If $u_1, v_1$ are adjacent vertices in $G_1 = (V_1, E_1)$.

If $u_1, v_1$ are adjacent vertices in $G_1 = (V_1, E_1)$. The graph $G_1 = (V_1, E_1)$ be a subtractdivisor cordialgraph therefore we get mn number subtractdivisor cordialgraph isomorphic to $G_1 = (V_1, E_1)$. Hence we get mn times of $G_1 = (V_1, E_1)$.

From case (i) and (ii), we get $\psi = \left\{ \left( \bigcup_{i=1}^{m} H_i \right) \cup \left( \bigcup_{j=1}^{n} H_j \right) \cup \left( \bigcup_{j=1}^{n} \left( H_{1j}, H_{2j}, \ldots H_{mj} \right) \right) \right\}$ this implies $|\psi| = \sum_{i=1}^{m} H_i + \sum_{j=1}^{n} H_j + \sum_{j=1}^{n} \sum_{i=1}^{m} H_{ij} = m + n + mn$. Hence we get $\pi_{SUB}(G_1 \circ G_2) \geq (m + n + mn)$.

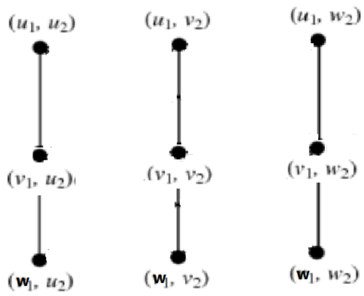
**Illustration 2.3:** The Cartesian product of two subtractdivisor cordialgraphs $P_3 \& P_3$ is given in Figure.2.3



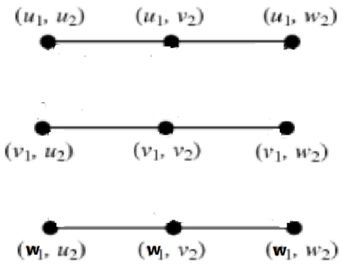




Decomposition of $G_1 \circ G_2$ $\qquad$ $G_1 \circ G_2$

Isomorphic to $G_1$




Isomorphic to $G_2$




Isomorphic to 'mn' times of $G_1$

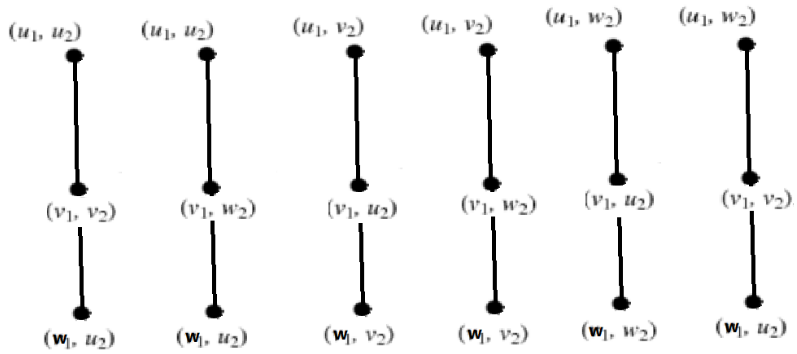



Figure.2.3

## 3. CONCLUSION

In this paper we define subtractdivisor decomposition and subtractdivisor number $\pi_{SUB}(G)$ of graphs. Also investigate some bounds of $\pi_{SUB}(G)$ in product graphs like Cartesian product, composition etc. In future we will investigate thedecompositionlabelingnumber various in graphs.

## 4. REFERENCES

[1] F. Harary, Graph Theory, Addison-wesley, Reading, Mass, (1972).
[2] A. Rosa, On certain valuation of graph, Theory of Graphs (Rome, July 1966), Goden and Breach, N. Y. and Paris, 1967, 349−355.
[3] C. Barrientos, The gracefulness of unions of cycles and complete bipartite graphs, J. Combin. Math. Combin. Comput. 52 (2005), 69 − 78.
[4] F. Harary, Graph theory Addition Wesley, Massachusetts, 1972.
[5] J. A. Bondy and U. S. R. Murty, Graph Theory with Applications, Macmillan Press, London, 1976.
[6] G. Chartrand, L. Lesniak, Graph and Digraphs, 3rd edition, Chapman & Hall, London, 1996.