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Analyzing The Performance Efficiency Of Rubber Industries In India: Two-Stage Fuzzy DEA Approach

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Abstract: The primary goal of this study is to use Two Stage Fuzzy Data Envelopment Analysis approach in Fuzzy environment to assess the efficiency of Rubber Industries functioning in India. The methodology proposes a framework for evaluating various Production and Marketability measures in the selected Industries in order to determine which are the excellent. By using a hybrid learning procedure, the proposed Fuzzy Inference System can construct an input-output mapping based on the form of fuzzy if-then rules and stipulated input-output data pairs. The analysis of the results shows that some Industries are efficient in terms of production, while others are efficient in terms of marketability.

Keywords: Fuzzy Inference System, Graded Mean Integration Representation, Fuzzy Constant Returns to Scale model, Fuzzy Variable Returns to Scale model.

1. INTRODUCTION

Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a methodology based upon an interesting application of linear programming. It was originally developed for performance measurement. It has been successfully employed for assessing the relative performance of a set of firms that use a variety of identical inputs to produce a variety of identical outputs. The principles of DEA date back to Farrell (1957) [5]. The recent series of discussions on this topic started with the article by Charnes et al (1978) [3].

Decision-Making Units

Data Envelopment Analysis is a linear programming-based technique for measuring the performance efficiency of organizational units which are termed Decision-Making Units (DMUs). This technique aims to measure how efficiently a DMU uses the resources available to generate a set of outputs (Charnes et al 1978) [3]. The performance of DMUs is assessed in DEA using the concept of efficiency or productivity, which is the ratio of total outputs to total inputs. Efficiencies estimated using DEA are relative, that is, relative to the best performing DMU (or DMUs if there is more than one best-performing DMUs) [9, 11]. The best performing DMU is assigned an efficiency score of unity or 100 per cent, and the performance of other DMUs varies, between 0 and 100 percent relative to this best performance [9, 11].

Two Stage DEA

Fare and Grosskopf (1996) [4], who introduced a DEA network approach that views DMU as a sub-system network. Within the context of DEA, there are a number of methods that have the potential to be used in efficiency evaluation. As part of the growth of the DEA network, a significant number of studies have been devoted to the two-stage DEA in the past few years. The two-stage DEA model produces separate performance measures for Stage 1 and Stage 2. There are two forms of the current two-stage DEA model: the two-stage closed DEA system and the two-stage open DEA system [4].

Two-stage Closed DEA System Model

In the two-stage closed DEA system model, the output measures of the first stage are considered to be the input measures of the second stage.

Two-stage Open DEA System Model

In addition to the intermediate variables, in the two-stage open DEA system model, the second stage has new inputs, so that the second stage inputs are not exactly the first stage outputs.

2. REVIEW OF LITERATURE

Two-stage DEA models can be applied in a various field, according to previous research. Färe and Grosskopf (1996) [4] proposed a DEA approach to the network, which treats the DMU as a network of sub-systems. As the DEA network has expanded in recent years, a large number of studies have been devoted to the two-stage DEA system. Seiford and Zhu, (1999) [12] examined the profitability and marketability of the top 55 U.S. commercial banks by applying the DEA model and concluded that large banks performed better with respect to profitability than small size banks, while small size banks have the better characteristic of marketability as compared to large size banks [12]. In 2014,MadjidTavana., et.al., [7] proposed an efficient two-stage fuzzy DEA model to calculate the efficiency scores for a DMU and its sub-DMUs and used the Stackelberg (leader–follower) game theory approach to prioritize and sequentially decompose the efficiency score of the DMU into a set of efficiency scores for its sub-DMUs.Peter Wanke., et.al.,analyzed the efficiency levels of the banking industry in the BRICS countries (Brazil, Russia, India, China, and South Africa) using an integrated two-stage fuzzy Approach in 2017 [10]. MadjidTavanaa,b.,et.al., (2018) [8] applied Fuzzy two-stage Game-DEA to a case study involving the assessment of sixty branches from the Saman bank in Iran and the approach leads to the overall enhancement of the efficiency scores through a cooperative game environment.

This article differs entirely from all other previous works by investigating and examining the current performance of the Rubber Industries functioning in India individually for the period [2016 – 2020] using Two Stage Data Envelopment Analysis. The productivity efficiency of the selected industries is assessed in the first stage, and the marketability of the industries is determined in the second stage using Returns to Scale Models.

3. RESEARCH METHODOLOGY

Data Collection

For this study, the required data of selected Rubber Industries based on the availability of reputed data have been taken from the Official Website of each industry for the financial years 2016–2020.

Selection of Input and Output Variables

Reviewing the literature on the application of DEA, different studies have used different combination of inputs and outputs. For the current study, the researcher considered five input variables and two output variables for Stage: 1 [Productivity] and two input variables and four output variables for Stage: 2 [Marketability] in order to have an elaborate study. The variables under the study are listed below:

Stage: 1

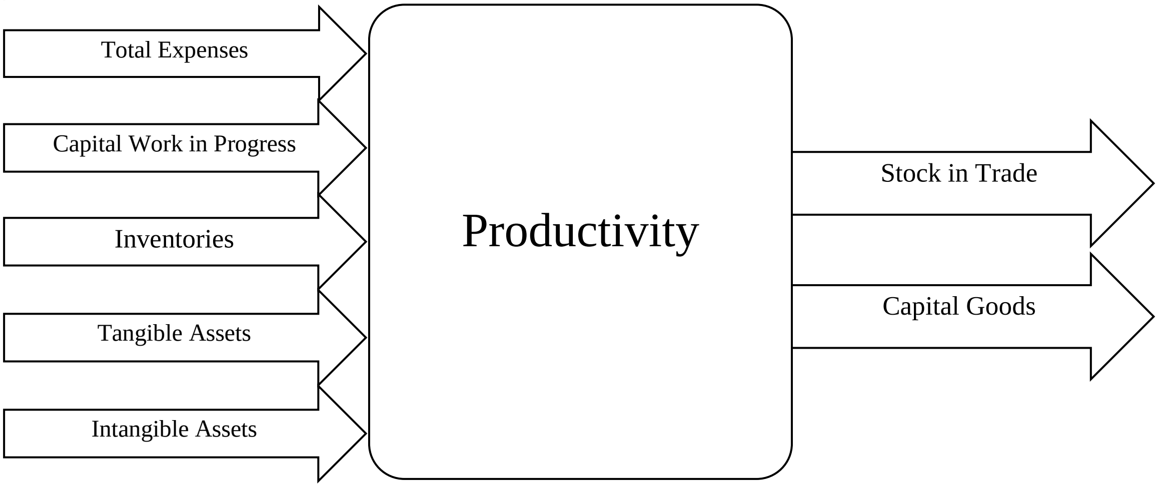


Figure: 1 Variables Considered in Stage 1 [Productivity]

Stage: 2

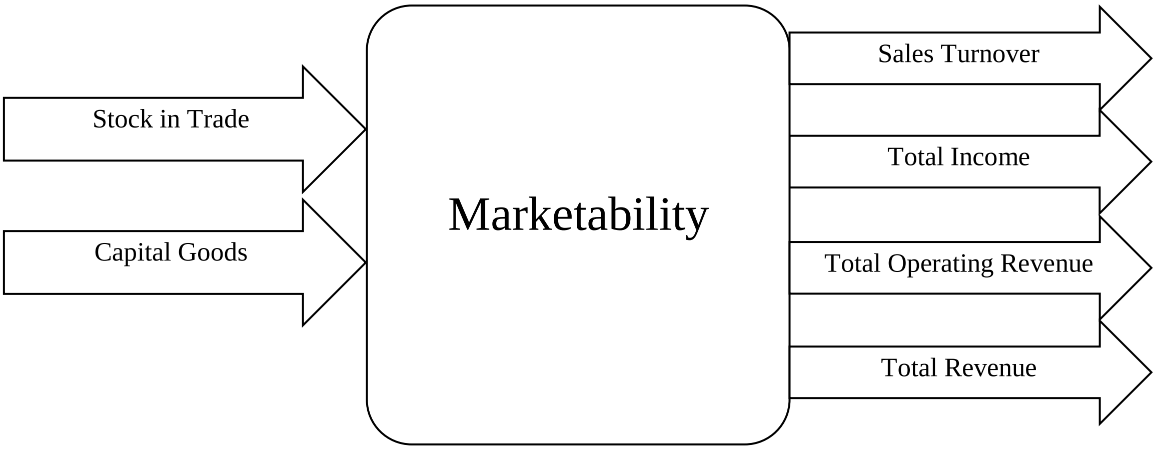


Figure: 2 Variables Considered in Stage 2 [Marketability]

Tangible Assets includes Land, Buildings, Plant & Machinery, Office Equipment, Vehicles where Intangible Assets includes Computer Software.

4. MATHEMATICAL MODELING

Fuzzy Inference System

A fuzzy inference system is composed of five functional blocks:

1. A rule base containing a number of fuzzy IF–THEN rules.

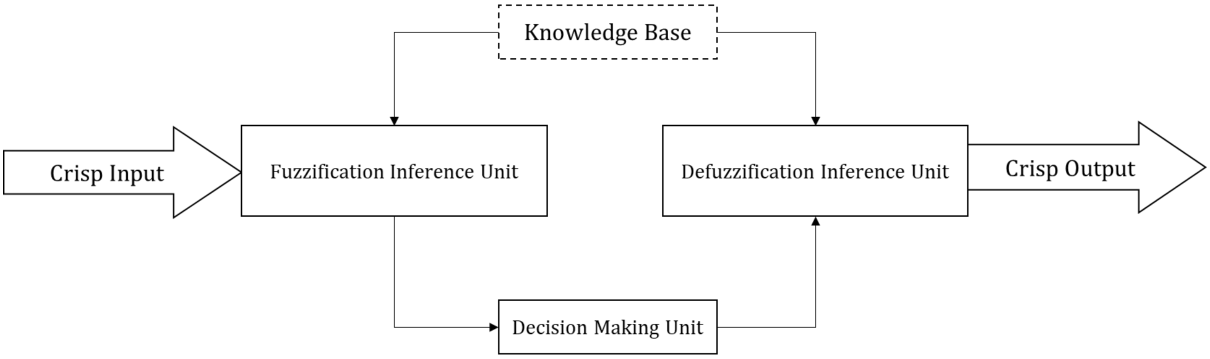
2. A database which defines the membership functions of the fuzzy sets used in the fuzzy rules.

3. A decision making unit which performs the inference operations on the rules.

4. A fuzzification inference which transforms the crisp inputs into degree of match with linguistic values.

5. A defuzzification interface which transforms the fuzzy results of the inference into a crisp output.

Usually, the rule base and the database are jointly referred to as the knowledge base. Several types of FIS have been proposed in the literature. It is due to the differences between the specification of the consequent part and the defuzzification schemes [13, 14]



Flowchart: 1 Fuzzy Inference System

The Fuzzy DEA principles:

The observed values in real-world problems are often imprecise or vague. Imprecise or vague data may be the result of unquantifiable, incomplete and non-obtainable information. Imprecise or vague data is often expressed with bounded intervals, ordinal (rank order) data or fuzzy numbers. In recent years, many researchers have formulated fuzzy DEA models to deal with situations where some of the input and output data are imprecise or vague [1].

Fuzzy Fractional DEA Program:

Let's compare N DMUs' efficiency and consider one of the DMUs as being m^{th}.DMU [3, 9, 11].

The mathematical problem is,

$$Max\ \tilde{E}_m = \frac{\sum_{j=1}^{J} v_{jm}\tilde{y}_{jm}}{\sum_{i=1}^{I} u_{im}\tilde{x}_{im}}$$

Subject to the Constraints

$$0 \leq \frac{\sum_{j=1}^{J} v_{jm}\tilde{y}_{jm}}{\sum_{i=1}^{I} u_{im}\tilde{x}_{im}} \leq 1; \quad \mathrm{n} = 1,2,\dots,\mathrm{k},\mathrm{j}$$

$$v_{jm}, u_{im} \geq 0; \;\; i = 1,2,\dots \mathrm{k},\mathrm{i}; \;\; j = 1,2,\dots k$$

Where,
$\tilde{E}_m$ is the efficiency of the m^{th}DMU,
$\tilde{v}_{ij}$is the j^{th} fuzzy output of the m^{th}DMU,
$y_{jm}$is the weight of that output,
$\tilde{u}_{im}$ is i^{th} the fuzzy input of the m^{th}DMU,
$x_{jm}$is the weight of that input and
$Y_{jn}$and X_{in} are output j^{th} and i^{th} input, respectively, of the n^{th} DMU, n = 1, 2, …, N.
Note that here n includes m. [3, 9, 11]

Fuzzy Constant Returns to Scale Model:
General Form of Output-Oriented F-CRS Model
Stage 1:
The structure of the Output Maximization F-DEA [F-CRS] model can be viewed in the form of Fractional Programming problem as follows [3, 9, 11]:

Here the general model is constructed to maximize the efficiency of the q^{th} output variable:

$\tilde{v}_{jq} - j^{th}$ fuzzy output value of the q^{th} DMU of stage 1

$y_{jq} - j^{th}$output variable of theq^{th} DMUof stage 1

$\tilde{u}_{iq} - i^{th}$ fuzzy input valueof the q^{th} DMUof stage 1

$x_{iq} - i^{th}$ input variable of the q^{th} DMUof stage 1

$\tilde{E}_q -$ Efficiency of the q^{th} DMUof stage 1

$$Max\ \tilde{E}_q = \frac{\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq}}{\sum_{i=1}^{s} \tilde{u}_{iq} x_{iq}}$$

Subject to the constraints

$$\frac{\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq}}{\sum_{i=1}^{s} \tilde{u}_{iq} x_{iq}} \leq 1; q = 1,2,\dots,n$$

$$v_{jq}, y_{jq}, \tilde{u}_{iq}, \tilde{x}_{iq} \geq 0\ for\ all\ i = 1,2,..s; j = 1,2,..m, q = 1,2,\dots n$$

The Equivalent Linear programming problem for the above fractional model can be defined as follows [3, 9, 11]:

$$\mathrm{Min}\, \tilde{E}_q = \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq}$$

Subject to the constraints

$$\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} = 1; \quad \sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} \; - \; \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq} \leq 0; \qquad q = 1,2,\dots,n$$

$$\tilde{v}_{jq}, y_{jq}, \tilde{u}_{iq}, x_{iq} \geq 0 \; for \; all \; i = 1,2,..s; j = 1,2,..m, q = 1,2,\dots n$$

Stage 2:

The general form of the Second Stage Output Maximization F-DEA [F-CRS] model can be expressed in the form of Fuzzy Fractional Programming Model as follows:

Here the general model is constructed to maximize the efficiency of the qth output variable:

$\tilde{w}_{jq} - j^{th}$ fuzzy output value of the q^{th} DMU of stage 2

$y_{jq} - j^{th}$ output variable of the q^{th} DMUof stage 2

$\tilde{v}_{iq} - i^{th}$ fuzzy input value of the q^{th} DMUof stage 2

$x_{iq} - i^{th}$ input variable of the q^{th} DMUof stage 2

$\tilde{E}_q -$ Efficiency of the q^{th} DMUof stage 2

$$\text{Max}\, \tilde{E}_q = \frac{\sum_{j=1}^{m} \tilde{w}_{jq} y_{jq}}{\sum_{i=1}^{s} \tilde{v}_{iq} x_{iq}}$$

Subject to the constraints

$$\frac{\sum_{j=1}^{m} \tilde{w}_{jq} y_{jq}}{\sum_{i=1}^{s} \tilde{v}_{iq} x_{iq}} \leq 1; q = 1,2,\dots,n$$

$$\tilde{w}_{jq}, y_{jq}, \tilde{v}_{iq}, x_{iq} \geq 0 \; for \; all \; i = 1,2,..s; j = 1,2,..m, q = 1,2,\dots n$$

The Equivalent Fuzzy Linear programming problem for the above fractional model can be defined as follows:

$$\text{Max}\, \tilde{E}_q = \sum_{j=1}^{m} w_{jq} \tilde{y}_{jq}$$

Subject to the constraints

$$\sum_{i=1}^{s} \tilde{v}_{iq} x_{iq} = 1$$

$$\sum_{j=1}^{m} \tilde{w}_{jq} y_{jq} - \sum_{i=1}^{s} \tilde{v}_{iq} x_{iq} \leq 0\,; \quad q = 1,2,\dots n$$

$$\tilde{w}_{jq}, y_{jq}, \tilde{v}_{iq}, x_{iq} \geq 0 \; for \; all \; i = 1,2,..s; j = 1,2,..m, q = 1,2,\dots n$$

General Form of Input-Oriented F-CRS Model

Stage 1:

The general form of Input Minimization F-DEA [F-CRS] Linear Programming model can be represented as follows [9, 11]:

$$\text{Min}\, \tilde{E}_q = \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq}$$

Subject to the constraints

$$\sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} = 1; \quad \sum_{j=1}^{m} \tilde{v}_{jq} y_{jq} - \sum_{i=1}^{s} \tilde{u}_{iq} x_{iq} \leq 0; \qquad q = 1,2,\dots,n$$

$$\tilde{v}_{jq}, y_{jq}, \tilde{u}_{iq}, x_{iq} \geq 0 \; for\ all\ i = 1,2,..s; j = 1,2,..m, q = 1,2,\dots n$$

Stage 2:

The general form of Second Stage Input Minimization F-DEA [F-CRS] Linear Programming model can be represented as follows:

$$\text{Min}\, \tilde{E}_q = \sum_{i=1}^{s} \tilde{v}_{iq} x_{iq}$$

Subject to the constraints

$$\sum_{j=1}^{m} \tilde{w}_{jq} y_{jq} = 1; \quad \sum_{j=1}^{m} \tilde{w}_{jq} y_{jq} - \sum_{i=1}^{s} \tilde{v}_{iq} x_{iq} \leq 0; \qquad q = 1,2,\dots,n$$

$$\tilde{w}_{jq}, y_{jq}, \tilde{v}_{iq}, x_{iq} \geq 0 \; for\ all\ i = 1,2,..s; j = 1,2,..m, q = 1,2,\dots n$$

Fuzzy Variable Returns to Scale Model:

Stage 1:

The DEA envelopment program for considering fuzzy variable return to scale referred by Banker et al. [2, 9, 11] is as follows:

$$\text{Min}\, \theta_m$$

Subject to the Constraints

$$\tilde{Y}\lambda \geq \tilde{Y}_m; \quad \tilde{X}\lambda \leq \theta\tilde{X}_m$$

$$\sum_{n=1}^{N} \lambda_n = 1;$$

$$\lambda \geq 0; \quad \theta_m \text{free variable}$$

Stage 2:

The general form of the Second Stage F-DEA for Variable Returns to Scale model is as follows [2, 9, 11]:

$$Min\, \theta_m$$

Subject to the Constraints

$$\tilde{X}\lambda \leq \tilde{X}_m; \quad \tilde{Z}\lambda \geq \theta\tilde{Z}_m$$

$$\sum_{n=1}^{N} \lambda_n = 1;$$

$$\lambda \geq 0; \quad \theta_m\ free\ variable$$

Pentagonal Fuzzy Number

A pentagonal fuzzy number, which represented with five points as follows, $A = (a_1, a_2, a_3, a_4, a_5)$, $a_i \in R$, This representation is interpreted as membership function μ_A

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \dfrac{x-a_2}{a_3-a_2} & a_2 \le x \le a_3 \\ 1 & x = a_3 \\ \dfrac{a_4-x}{a_4-a_3} & a_3 \le x \le a_4 \\ \dfrac{a_5-x}{a_5-a_4} & a_4 \le x \le a_5 \\ 0 & \text{otherwise} \end{cases}$$

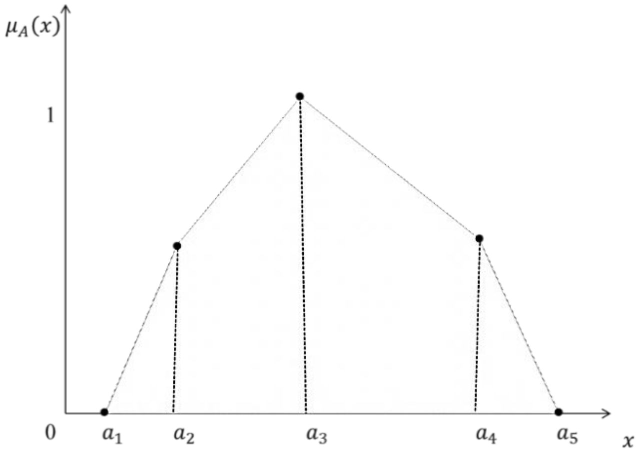


Figure: 3 Pentagonal Fuzzy Number

Defuzzification

Since technical processes require clear control actions, a procedure which generates net values from one or several given fuzzy numbers.

Graded Mean Integration representation

Chen and Hseih propose graded mean integration representation for representing generalized fuzzy number [13, 14].

If the generalized fuzzy number $A = (a_1, a_2, a_3, a_4, a_5 : w)$, then the graded mean h-level is$\frac{h[L^{-1}(h)+R^{-1}(h)]}{2}$. Where $L^{-1}$and R^{-1} are inverse functions of L and R.

And, the defuzzified value of the Fuzzy number A by the graded mean integration representation $\Re(A)$is defined as [13, 14]

$$\Re(A) = \frac{\int_0^h \left[\frac{L^{-1}(h)+R^{-1}(h)}{2}\right] dh}{\int_0^w h\, dh}$$

Where $h \in (0, w)$, and $0 < w \le 1$.

If $A = (a_1, a_2, a_3, a_4, a_5)$ is a pentagonal fuzzy number. Chen and Hsieh have already found the general formulae of the representation of generalized pentagonal fuzzy number as follows [13, 14]:

$$\Re(A) = \frac{a_1 + 3a_2 + 4a_3 + 3a_4 + a_5}{12}$$

For this study, the researcher used Graded Mean Integration representation method for defuzzification [13, 14].

Problem Formulation: Productivity Stage

Fuzzy Constant Returns to Scale [Output Maximization]

Apollo Tyres (2016)

$$Max\ E_{Apollo} = \frac{224.4x_1 + 173.77x_2}{7546.44x_3 + 416.37x_4 + 1019.75x_5 + 3286.9x_6 + 12.93x_7}$$

Subject to the Constraints,

$$\frac{5.14x_1 + 31.86x_2}{2356.97x_3 + 231.07x_4 + 286.4x_5 + 2849.6x_6 + 4.16x_7} \leq 1$$
$$\frac{138.16x_1 + 300.63x_2}{4706.59x_3 + 213.45x_4 + 619.25x_5 + 1904.9x_6 + 57.66x_7} \leq 1$$
$$\frac{25.66x_1 + 49.46x_2}{79.59x_3 + 0.26x_4 + 8.25x_5 + 10.95x_6 + 0.13x_7} \leq 1$$
$$\frac{410.46x_1 + 6.34x_2}{1557.7x_3 + 28.9x_4 + 128.61x_5 + 220.37x_6 + 0.07x_7} \leq 1$$
$$\frac{54.45x_1 + 74.64x_2}{304.81x_3 + 1.96x_4 + 33.14x_5 + 129.71x_6 + 0.67x_7} \leq 1$$
$$\frac{6.5x_1 + 35.09x_2}{317.41x_3 + 0.83x_4 + 22.19x_5 + 423.24x_6 + 0.52x_7} \leq 1$$
$$\frac{0.23x_1 + 16.29x_2}{206.82x_3 + 4.29x_4 + 29.35x_5 + 27.45x_6 + 0.06x_7} \leq 1$$
$$\frac{32.7x_1 + 135.46x_2}{4875.6x_3 + 88.92x_4 + 739.68x_5 + 3327.4x_6 + 6.04x_7} \leq 1$$
$$\frac{0.42x_1 + 1.16x_2}{17.69x_3 + 80.99x_4 + 4.14x_5 + 0.69x_6 + 0.47x_7} \leq 1$$
$$\frac{2.53x_1 + 1.37x_2}{19.75x_3 + 4.98x_4 + 0.75x_5 + 12.68x_6 + 0.04x_7} \leq 1$$
$$\frac{38.89x_1 + 740.17x_2}{15537.21x_3 + 1058.4x_4 + 1879.74x_5 + 4584.74x_6 + 9.06x_7} \leq 1$$
$$\frac{6.77x_1 + 2.25x_2}{57.1x_3 + 0.08x_4 + 12.81x_5 + 7.8x_6 + 0.8x_7} \leq 1$$
$$\frac{1.06x_1 + 8.54x_2}{185.21x_3 + 6.21x_4 + 33.32x_5 + 169.53x_6 + 0.48x_7} \leq 1$$
$$\frac{7.9x_1 + 8.54x_2}{148.65x_3 + 0.27x_4 + 7.2x_5 + 27.77x_6 + 0.02x_7} \leq 1$$
$$\frac{3.77x_1 + 35.58x_2}{1753.45x_3 + 43.09x_4 + 207.93x_5 + 359.03x_6 + 2.76x_7} \leq 1$$
$$\frac{0.37x_1 + 9.05x_2}{60.88x_3 + 0.01x_4 + 11.21x_5 + 15.24x_6 + 0.26x_7} \leq 1$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

The corresponding LPP structure for the above problem can be written as follows,

$$Max\ E_{ADMC} = 224.4x_1 + 173.77x_2$$

Subject to the Constraints,

$$7546.44x_3 + 416.37x_4 + 1019.75x_5 + 3286.9x_6 + 12.93x_7 = 1$$
$$5.14x_1 + 31.86x_2 - 2356.97x_3 - 231.07x_4 - 286.4x_5 - 2849.6x_6 - 4.16x_7 \leq 0$$
$$138.16x_1 + 300.63x_2 - 4706.59x_3 - 213.45x_4 - 619.25x_5 - 1904.9x_6 - 57.66x_7 \leq 0$$
$$25.66x_1 + 49.46x_2 - 79.59x_3 - 0.26x_4 - 8.25x_5 - 10.95x_6 - 0.13x_7 \leq 0$$
$$410.46x_1 + 6.34x_2 - 1557.7x_3 - 28.9x_4 - 128.61x_5 - 220.37x_6 - 0.07x_7 \leq 0$$
$$54.45x_1 + 74.64x_2 - 304.81x_3 - 1.96x_4 - 33.14x_5 - 129.71x_6 - 0.67x_7 \leq 0$$
$$6.5x_1 + 35.09x_2 - 317.41x_3 - 0.83x_4 - 22.19x_5 - 423.24x_6 - 0.52x_7 \leq 0$$
$$0.23x_1 + 16.29x_2 - 206.82x_3 - 4.29x_4 - 29.35x_5 - 27.45x_6 - 0.06x_7 \leq 0$$
$$32.7x_1 + 135.46x_2 - 4875.6x_3 - 88.92x_4 - 739.68x_5 - 3327.4x_6 - 6.04x_7 \leq 0$$
$$0.42x_1 + 1.16x_2 - 17.69x_3 - 80.99x_4 - 4.14x_5 - 0.69x_6 - 0.47x_7 \leq 0$$

$$2.53x_1 + 1.37x_2 - 19.75x_3 - 4.98x_4 - 0.75x_5 - 12.68x_6 - 0.04x_7 \leq 0$$
$$38.89x_1 + 740.17x_2 - 15537.21x_3 - 1058.4x_4 - 1879.74x_5 - 4584.74x_6 - 9.06x_7 \leq 0$$
$$6.77x_1 + 2.25x_2 - 57.1x_3 - 0.08x_4 - 12.81x_5 - 7.8x_6 - 0.8x_7 \leq 0$$
$$1.06x_1 + 8.54x_2 - 185.21x_3 - 6.21x_4 - 33.32x_5 - 169.53x_6 - 0.48x_7 \leq 0$$
$$7.9x_1 + 8.54x_2 - 148.65x_3 - 0.27x_4 - 7.2x_5 - 27.77x_6 - 0.02x_7 \leq 0$$
$$3.77x_1 + 35.58x_2 - 1753.45x_3 - 43.09x_4 - 207.93x_5 - 359.03x_6 - 2.76x_7 \leq 0$$
$$0.37x_1 + 9.05x_2 - 60.88x_3 - 0.01x_4 - 11.21x_5 - 15.24x_6 - 0.26x_7 \leq 0$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

Fuzzy Variable Returns to Scale [Output Maximization]

Apollo Tyres (2016)

Min $x_{18} - x_{19}$

Subject to the constraints

$$\begin{aligned}224.4x_1 &+ 5.14x_2 + 138.16x_3 + 25.66x_4 + 410.46x_5 + 54.45x_6 + 6.5x_7 + 0.23x_8 \\ &+ 32.7x_9 + 0.42x_{10} + 2.53x_{11} + 38.89x_{12} + 6.77x_{13} + 1.06x_{14} + 7.9x_{15} \\ &+ 3.77x_{16} + 0.37x_{17} \geq 224.4\end{aligned}$$

$$\begin{aligned}173.77x_1 &+ 31.86x_2 + 300.63x_3 + 49.46x_4 + 6.34x_5 + 74.64x_6 + 35.09x_7 + 16.29x_8 \\ &+ 135.46x_9 + 1.16x_{10} + 1.37x_{11} + 740.17x_{12} + 2.25x_{13} + 8.54x_{14} \\ &+ 8.54x_{15} + 35.58x_{16} + 9.05x_{17} \geq 173.77\end{aligned}$$

$$\begin{aligned}7546.44x_1 &+ 2356.97x_2 + 4706.59x_3 + 79.59x_4 + 1557.7x_5 + 304.81x_6 + 317.41x_7 \\ &+ 206.82x_8 + 4875.6x_9 + 17.69x_{10} + 19.75x_{11} + 15537.21x_{12} + 57.1x_{13} \\ &+ 185.21x_{14} + 148.65x_{15} + 1753.45x_{16} + 60.88x_{17} + 7546.44x_{18} \\ &- 7546.44x_{19} \leq 0\end{aligned}$$

$$\begin{aligned}416.37x_1 &+ 231.07x_2 + 213.45x_3 + 0.26x_4 + 28.9x_5 + 1.96x_6 + 0.83x_7 + 4.29x_8 \\ &+ 88.92x_9 + 80.99x_{10} + 4.98x_{11} + 1058.4x_{12} + 0.08x_{13} + 6.21x_{14} \\ &+ 0.27x_{15} + 43.09x_{16} + 0.01x_{17} + 416.37x_{18} - 416.37x_{19} \leq 0\end{aligned}$$

$$\begin{aligned}1019.75x_1 &+ 286.4x_2 + 619.25x_3 + 8.25x_4 + 128.61x_5 + 33.14x_6 + 22.19x_7 + 29.35x_8 \\ &+ 739.68x_9 + 4.14x_{10} + 0.75x_{11} + 1879.74x_{12} + 12.81x_{13} + 33.32x_{14} \\ &+ 7.2x_{15} + 207.93x_{16} + 11.21x_{17} + 1019.75x_{18} - 1019.75x_{19} \leq 0\end{aligned}$$

$$\begin{aligned}3286.9x_1 &+ 2849.6x_2 + 1904.9x_3 + 10.95x_4 + 220.37x_5 + 129.71x_6 + 423.24x_7 \\ &+ 27.45x_8 + 3327.4x_9 + 0.69x_{10} + 12.68x_{11} + 4584.74x_{12} + 7.8x_{13} \\ &+ 169.53x_{14} + 27.77x_{15} + 359.03x_{16} + 15.24x_{17} + 3286.9x_{18} \\ &- 3286.9x_{19} \leq 0\end{aligned}$$

$$\begin{aligned}12.93x_1 &+ 4.16x_2 + 57.66x_3 + 0.13x_4 + 0.07x_5 + 0.67x_6 + 0.52x_7 + 0.06x_8 + 6.04x_9 \\ &+ 0.47x_{10} + 0.04x_{11} + 9.06x_{12} + 0.8x_{13} + 0.48x_{14} + 0.02x_{15} + 2.76x_{16} \\ &+ 0.26x_{17} + 12.93x_{18} - 12.93x_{19} \leq 0\end{aligned}$$

$$\sum_{i=1}^{19} x_i = 1$$
$$x_i \geq 0, i = 1,2,\dots,19$$

All Such 340 problems were generated from the collected data and solved using the software TORA.

5. EMPIRICAL RESULTS

Stage: 1 [Productivity]

Fuzzy Constant Return to Scale [F-CCR Model]

The F-DEA efficiency score is based on Technical Efficiency [Fuzzy Constant return to scale] under the F-CCR Model, as shown in Table 1. According to the study based on the F-

CRS Model, only two industries achieved full efficiency score 1 in productivity for the financial years 2016–2020.

Table 1: Technical Efficiency Result on Productivity under F-CRS Model

DMUs	**Efficiency Score**	**De-fuzzified Score**
Apollo Tyres	0.092,0.182,0.132,0.238,0.272	0.1793
Balkrishna Industries	0.022,0.053,0.205,0.238,0.078	0.1494
CEAT	0.103,0.096,0.102,0.117,0.064	0.1012
Eastern Treads	1,1,1,1,1	1
Goodyear India	1,1,1,1,1	1
GRP	0.554,0.517,1,1,1	0.8421
Harrisons Malayalam	0.264,0.252,0.408,0.603,0.287	0.3957
INDAG Rubber	0.689,0.529,1,0.182,0.135	0.5798
JK Tyre & Industries	0.059,0.34,0.484,0.835,0.466	0.4988
MM Rubber Company	0.372,0.095,0.152,0.104,1	0.2148
Modi Rubber	1,1,0.824,1,1	0.9413
MRF	0.211,0.035,0.085,0.036,0.058	0.0685
Multibase India	0.857,1,1,1,0.9	0.9798
PIX Transmissions	0.074,0.058,0.163,0.152,0.097	0.1211
RUBFILA International	1,1,0.766,0.463,1	0.7878
TVS Sri chakra	0.034,0.03,0.061,0.031,0.018	0.0399
Vamshi Rubber	1,0.25,1,0.323,0.071	0.5658

Fuzzy Variable Return to Scale [F-BCC Model]

The F-DEA performance score based on Technical Efficiency [Fuzzy Variable Return to Scale] under the F-BCC Model is communicated in Table 2 Based on the F-VRS model, 8 industries achieved the highest productivity efficiency score among the selected industries for the financial years 2016-2020.

Table 2: Technical Efficiency Result on Productivity under F-VRS Model

DMUs	**Efficiency Score**	**De-fuzzified Score**
Apollo Tyres	0.506,1,1,0.39,1	0.8063
Balkrishna Industries	0.024,0.24,1,1,0.304	0.6707
CEAT	1,1,1,1,0.79	0.9825
Eastern Treads	1,1,1,1,1	1
Goodyear India	1,1,1,1,1	1
GRP	1,1,1,1,1	1
Harrisons Malayalam	0.353,0.322,0.417,0.658,0.443	0.4503
INDAG Rubber	0.788,1,1,0.577,0.546	0.8388
JK Tyre & Industries	0.972,1,1,1,1	0.9977
MM Rubber Company	1,1,1,1,1	1
Modi Rubber	1,1,1,1,1	1
MRF	1,0.342,1,0.536,0.965	0.7166
Multibase India	1,1,1,1,1	1
PIX Transmissions	0.289,0.425,0.456,0.336,0.219	0.3846
RUBFILA International	1,1,1,1,1	1

TVS Sri chakra	0.037,0.039,0.061,0.039,0.033	0.0457
Vamshi Rubber	1,1,1,1,1	1

Overall Mean Efficiency

There are only two Industries that are highly consistent with an efficiency score of 1 and rank first in productivity among all the Rubber Industries considered. Table 3 shows the mean of mean efficiency of Rubber Industry in stage 1.

Table 3: Overall Technical Efficiency Result on Productivity

DMUs	**CRS Score**	**VRS Score**	**Mean Score**
Apollo Tyres	0.1793	0.8063	0.49
Balkrishna Industries	0.1494	0.6707	0.41
CEAT	0.1012	0.9825	0.54
Eastern Treads	1	1	1
Goodyear India	1	1	1
GRP	0.8421	1	0.92
Harrisons Malayalam	0.3957	0.4503	0.42
INDAG Rubber	0.5798	0.8388	0.71
JK Tyre & Industries	0.4988	0.9977	0.75
MM Rubber Company	0.2148	1	0.61
Modi Rubber	0.9413	1	0.97
MRF	0.0685	0.7166	0.39
Multibase India	0.9798	1	0.99
PIX Transmissions	0.1211	0.3846	0.25
RUBFILA International	0.7878	1	0.89
TVS Sri chakra	0.0399	0.0457	0.04
Vamshi Rubber	0.5658	1	0.78

Stage: 2 [Marketability]

Fuzzy Constant Return to Scale [F-CCR Model]

The F-DEA performance score based on Technical Efficiency [Fuzzy Constant return to scale] under the F-CCR Model is communicated in Table 4 Based on the F-CRS Model, only one industry achieved maximum efficiency score 1 in marketability for the financial years 2016 – 2020.

Table 4: Technical Efficiency Result on Marketability under F-CRS Model

DMUs	**Efficiency Score**	**De-fuzzified Score**
Apollo Tyres	0.526,0.159,0.383,1,0.191	0.4772
Balkrishna Industries	1,0.753,0.393,0.355,0.945	0.5701
CEAT	0.197,0.295,0.336,0.276,0.312	0.2972
Eastern Treads	0.018,0.026,0.031,0.029,0.06	0.0306
Goodyear India	1,1,1,1,1	1
GRP	0.042,0.069,0.08,0.095,0.107	0.0801
Harrisons Malayalam	0.08,0.177,0.209,0.173,0.258	0.1853
INDAG Rubber	1,0.595,0.815,0.459,0.392	0.6512

JK Tyre & Industries	0.468,0.449,0.441,0.5,0.679	0.4798
MM Rubber Company	0.176,0.284,0.366,0.466,0.396	0.3572
Modi Rubber	0.159,0.1,0.203,0.218,0.519	0.2037
MRF	0.777,1,0.581,1,0.334	0.7863
Multibase India	0.312,0.287,0.354,0.498,0.275	0.3632
PIX Transmissions	0.319,0.556,0.39,0.274,0.384	0.3961
RUBFILA International	0.192,0.452,0.766,0.524,0.611	0.5663
TVS Sri chakra	0.894,1,1,1,1	0.9912
Vamshi Rubber	0.257,0.137,0.307,0.2,0.267	0.2303

Fuzzy Variable Return to Scale [F-BCC Model]

The F-DEA performance score based on Technical Efficiency [Fuzzy Variable Return to Scale] under the F-BCC Model is communicated in Table 5 Based on the F-VRS model, four industries achieved the highest efficiency score in marketability among the selected industries for the financial years 2016-2020.

Table 5: Technical Efficiency Result on Marketability under F-VRS Model

DMUs	**Efficiency Score**	**De-fuzzified Score**
Apollo Tyres	1,0.159,0.646,1,0.5	0.6301
Balkrishna Industries	1,1,0.577,0.355,1	0.6978
CEAT	0.378,0.296,0.53,0.28,0.766	0.4160
Eastern Treads	0.037,0.041,0.051,0.071,0.381	0.0798
Goodyear India	1,1,1,1,1	1
Grp	0.053,0.08,0.09,0.103,0.12	0.0902
Harrisons Malayalam	0.121,0.201,0.229,0.188,0.283	0.2073
INDAG Rubber	1,1,1,1,1	1
JK Tyre & Industries	1,0.449,0.772,0.5,1	0.6613
MM Rubber Company	1,1,1,1,1	1
Modi Rubber	0.86,0.311,0.601,1,1	0.6831
MRF	1,1,1,1,1	1
Multibase India	0.699,0.456,0.49,0.682,0.492	0.5471
PIX Transmissions	0.593,0.682,0.552,0.302,0.436	0.5158
RUBFILA International	0.297,0.582,0.906,0.603,0.74	0.6847
TVS Sri chakra	0.911,1,1,1,1	0.9926
Vamshi Rubber	0.899,1,1,0.972,1	0.9846

Overall Mean Efficiency

There is only one Industry that is highly consistent with an efficiency score of 1 and ranks first among all the Rubber considered. The mean of mean efficiency of the Rubber Industries are given in the Table 6.

Table 6: Overall Technical Efficiency Result on Marketability

DMUs	**CRS Score**	**VRS Score**	**Mean Score**
Apollo Tyres	0.4772	0.6301	0.55
Balkrishna Industries	0.5701	0.6978	0.63

CEAT	0.2972	0.4160	0.36
Eastern Treads	0.0306	0.0798	0.06
Goodyear India	1	1	1
Grp	0.0801	0.0902	0.09
Harrisons Malayalam	0.1853	0.2073	0.20
INDAG Rubber	0.6512	1	0.83
JK Tyre & Industries	0.4798	0.6613	0.57
MM Rubber Company	0.3572	1	0.68
Modi Rubber	0.2037	0.6831	0.44
MRF	0.7863	1	0.89
Multibase India	0.3632	0.5471	0.46
PIX Transmissions	0.3961	0.5158	0.46
RUBFILA International	0.5663	0.6847	0.63
TVS Sri chakra	0.9912	0.9926	0.99
Vamshi Rubber	0.2303	0.9846	0.61

6. CONCLUSION

The researcher analysed the performance of the industries in two separate units, Productivity and Marketability. The two-stage Data Envelopment Analysis shows that in terms of production Eastern Treads & Goodyear India are the most efficient industries, although in terms of marketability only Goodyear India achieved the efficiency standard. In addition, the study reveals that some industries are efficient in terms of productivity, while others are efficient in terms of marketability.

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